MAT 295 Calculus I
Final Examination
Spring 2014

Print your name ____________________________

Signature ____________________________

SU ID number ________________

Print your instructor's name ____________________________

Instructions. This examination has 12 problems and 11 pages (including this one). Make sure your examination is complete before you begin work.

This examination is worth 150 points. The point values are indicated for each of the problems.

Show your work clearly. All answers must be justified.

The use of or availability of any electronic device, notes or books during this exam is a violation of the Academic Integrity Policy.

Do NOT write below this line!

1. _____ 7. _____
2. _____ 8. _____
3. _____ 9. _____
4. _____ 10. _____
5. _____ 11. _____
6. _____ 12. _____

TOTAL ________ = ________%
(1) [20 pts] Evaluate each of the following limits. Write DNE if the limit does not exist.

(a) \( \lim_{x \to 0} \frac{\tan 3x}{x + \sin x} \)

(b) \( \lim_{x \to 25} \frac{5 - \sqrt{x}}{25 - x} \)

(c) \( \lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^x \)
(2) [18 pts] Find the derivatives of the following functions. Do NOT simplify your answers.

(a) \( y = \sec^2 x + \cos(x^2) \)

(b) \( y = e^{sx} \arcsin(3x) \)

(c) \( g(t) = \frac{\ln(t)}{1+t^2} \)
(3) [8 pts] Find $\frac{dy}{dx}$ if $y = (\cos x)^x$

(4) [12 pts] Find $\frac{dy}{dx}$ if $y \sin(x) = 3x^2 + 2y^3$. 
(5) [10 pts]

If $f$ is continuous at $a$, then $f$ is differentiable at $a$.

TRUE  FALSE

Justify your answer:

If $f'(c) = 0$, then $f$ has a local maximum or minimum at $c$.

TRUE  FALSE

Justify your answer:
(6) [15 pts] A kite 100 feet above the ground moves horizontally at a speed of 10 ft/sec. At what rate is the angle between the string and the horizontal decreasing when 200 feet of string have been let out?

Include a carefully labeled sketch as part of your solution. Leave your answer as an exact value. Include the units for your answer.
(7) [9 pts] The graph of \( y = f'(x) \) on the interval \([-2, 8]\) is shown below.

(a) On what interval(s) is \( f \) decreasing? Explain.

(b) On what interval(s) is \( f \) concave up? Explain.

(c) At what value(s) of \( x \), if any, does \( f \) have a local maximum? Explain.
(8) [8 pts] Find the intervals of concavity and the coordinates of the points of inflection (if any) of \( f(x) = x^4 - 4x^3 + 1 \).

Justify your answers.
(9) [15 pts] A box with a square base and open top must have a volume of 32 cubic centimeters. Find the dimensions of the box that minimize the amount of material used.

Include a carefully labeled sketch as part of your solution. Justify your solution using the appropriate test(s). Include the units for your answer.
(10) [7 pts] An oil storage tank ruptures at time \( t = 0 \) and oil leaks from the tank at a rate of \( r(t) = 200e^{-0.1t} \) liters per minute. How much oil leaks out in the first 2 hours? Leave your answer as an exact value and include appropriate units.

(11) [8 pts] If \( f(x) = \frac{1}{x} \) on the interval \( 1 \leq x \leq 7 \), evaluate the Riemann sum with \( n = 3 \), taking the sample points to be the left endpoints. Support your answer with a graph.
Evaluate the following integrals.

(a) \[ \int \frac{\sqrt{x} - x^2 + 1}{x} \, dx \]

(b) \[ \int \frac{x}{x^2 + 9} \, dx \]

(c) \[ \int e^{\cos x} \sin x \, dx \]

(d) \[ \int \frac{\cos x}{\sin^4 x} \, dx \]