MAT 286
May 7, 2012
Final Exam

Show your work. No work-no credit!

Name ____________ SU ID# ____________

Please circle your instructor’s NAME.
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Eric Ottman, Zeynep Ozal, Hamidreza Rahmati

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1. Find the following integrals. [5 points for each problem.]

(a) \[ \int (4x^3 + 6x - 5) \, dx \]

(b) \[ \int 4e^{-3x} \, dx \]
(c) \[ \int \frac{x}{x^2 + 5} \, dx \]

(d) \[ \int x \sin x \, dx \]
2. For (a) and (b), find the values of the definite integrals. For (c) and (d), each of the improper integrals, determine if it converges or diverges, and find the value of each that converges. [5 points for each problem.]

(a) \( \int_{-1}^{8} (x^3)dx \)

(b) \( \int_{\frac{x}{e}}^{2x} \cos x \, dx \)
(c) \[ \int_{-\infty}^{\infty} \frac{x}{x^2 + 5} \, dx \] [compare with 1(c).]

(d) \[ \int_{0}^{\infty} \frac{2x}{(x^2 + 1)^2} \, dx \]
3. (a) Find the average value of the function, \( f(x) = \sqrt{x + 1} \), over the interval \([3, 8]\). [5 points]

(b) Find the volume under the given surface,

\[
 f(x, y) = (4 - x^2) y,
\]

and above the rectangle with the given boundaries, \(-2 \leq x \leq 2, 0 \leq y \leq 3\). [5 points]
4. (a) For \( f(x, y) = x^2 + y^2 + xy + 6x - 5 \),
find values of \( x \) and \( y \) such that both \( f_x(x, y) = 0 \) and \( f_y(x, y) = 0 \). [5 points]

(b) For \( f(x, y, z) = \ln |x^2 - 5xz^3 + y^4| \),
find \( f_z(x, y, z) \), and \( f_{yz}(x, y, z) \). [5 points]
5. (a) Evaluate the following integral [5 points]

\[ \int_{4}^{5} x \sqrt{x^2 + 3y} \, dy. \]

(b) Find the double integral over the rectangular region \( \mathcal{R} \) with given boundaries. [5 points]

\[ \iint_{\mathcal{R}} (4x - 3y^2) \, dx \, dy; \quad 0 \leq x \leq 2, \quad 1 \leq y \leq 5. \]
6. Solve the following differential equations. (Solve the initial value problems if the initial values are given, otherwise find the general solutions.) [5 points for each problem.]

(a) \( \frac{dy}{dx} = \frac{2x}{3y^2} \).

(b) \( \frac{dy}{dx} = \frac{1}{y}, \quad y = -3, \) when \( x = 0 \).
(c) \( \frac{dy}{dx} + 3y = e^{-2x} \).

(d) \( x \frac{dy}{dx} + 5y = x^3 \), for \( x > 0 \); with the initial condition \( y(1) = 2 \).
7. Use Euler's method to approximate the following function value to three decimal places, using $\Delta x = 0.1$ (or $h = 0.1$.)

$$\frac{dy}{dx} = x + y^2 \ ; \ y(0) = 1 \ ; \ find \ y(0.4)$$

[10 points]