Final Exam
MAT 296
Spring 2016

Circle your instructor's name:

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READ THIS BEFORE YOU BEGIN

This examination contains 12 problems on 12 pages (including this cover page). Point values are indicated. It is your responsibility to make sure that all problems and pages are present. A graphing calculator (TI83/84 or equivalent) is allowed on this exam. However, no symbolic-capable calculators (TI-nspire, TI89/92 or equivalent), books, or notes are allowed on this exam. Cell phones and other similar devices may not be out during the exam. Please silence them and put them away now. Your solutions must be written legibly and contain all of the necessary steps which enabled you to arrive at your answer to receive full credit for the problem. Unsupported answers will receive little or no credit. Circle your final answer.

DO NOT WRITE IN THE TABLE BELOW

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1. Evaluate the following integral.

\[ \int x^3 \ln x \, dx \]

(10 points)
2. Evaluate the following integral.

\[ \int \frac{x^3}{\sqrt{1 + x^2}} \, dx \]

(10 points)
3. Evaluate the following integral.

\[
\int \frac{x^2 + 11x + 3}{(x - 1)^2(x + 4)} \, dx
\]

(10 points)
4. Let $R$ be the first quadrant region enclosed by the curves $y = x^2$ and $y = x^{1/3}$. Find the volume of the solid formed by revolving $R$ about the $y$-axis.

(10 points)
5. Set up, **but don’t evaluate**, an integral that gives the area of the surface formed by revolving the curve \( y = \cos \left( \frac{x}{2} \right) \) from \( x = 0 \) to \( x = \pi \) around the \( x \)-axis. You do not need to simplify the integral. Circle your final integral.

(5 points)

6. For the following geometric series, determine whether it converges or diverges. If the series converges, give the sum.

\[
\sum_{n=0}^{\infty} \frac{(-2)^n 3^{n+1}}{7^{n+1}}
\]

(5 points)
7. Determine whether the following improper integral converges or diverges. If it converges, give the value of the integral.

\[ \int_{e}^{\infty} \frac{1}{x(\ln x)^2} \, dx \]

(10 points)
8. Determine whether each series is convergent or divergent. Say which and state the name of each test you use and show that it applies.

(5 points each part)

a)
\[ \sum_{n=0}^{\infty} \frac{n^2 4^n}{n!} \]

b)
\[ \sum_{n=0}^{\infty} \frac{(-1)^n(5n + 1)}{12n - 5} \]
9. Find the interval of convergence for the power series

\[ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n} \, 5^n} \, (x - 2)^n. \]

Circle your answer.

(10 points)
10. Either from a series you know or from the definition, find the Maclaurin series for the function
\[ f(x) = \frac{1}{(1 - 2x)^2}. \]
You may write your answer in the form
\[ \text{term1 + term2 + term3 + term4 + ...} \]
(do not include terms that are 0), or with summation notation.
Simplify the coefficients.

(5 points)
11. Here is the Maclaurin series for $f(x) = \ln(1 + x^2)$:

$$\ln(1 + x^2) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^{2n}.$$ 

Use the series to estimate $\int_0^{1/3} \ln(1 + x^2) \, dx$ to within $\frac{1}{100000}$ (i.e. $10^{-6}$). Show your work and be sure to justify that your estimate has the required accuracy. Circle your estimate.

(5 points)
12. Use an integral to find the area inside one of the loops of

\[ r = 3 \cos 2\theta. \]

You must show your work, including the calculus necessary to justify a correct answer. You may graph the curve with your calculator.

(10 points)