MAT 183 Fall 2016  FINAL EXAM

Instructions:

• There are 14 problems on 15 pages.

• The only calculators allowed are TI-83, TI-83 Plus, TI-84 & TI-84 Plus.

• You may not use a calculator on your phone.

• You may not share calculators.

• Silence and put away all cell phones and similar electronic devices, including earbuds.

• Your solutions must be legible and contain intermediate steps. Unsupported answers will receive little or no credit.

• Write your answers in the space provided.

• Include all relevant work, but you may do messy calculations on scratch paper. If you want scratch paper, raise your hand and some will be brought to you. Turn in any scratch paper with your exam, but it will not be graded.

• Follow all instructions to receive full credit. In particular, follow instructions regarding rounding, decimal places and fractions.

• Write down your calculator commands.

Have a great Holiday Break!

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1. (10 points) Find all solutions to the following systems of linear equations. Write down intermediate steps and commands. Use fractions, not decimals, in your solutions.

(a) 
\[ \begin{align*} 
3x + 2y - z &= 3 \\
2x - y + 4z &= -1 \\
x + 3y - 5z &= 2 
\end{align*} \]

(b) 
\[ \begin{align*} 
x - y - z &= 4 \\
2x + 2y + 3z &= -1 \\
4x + z &= 7 
\end{align*} \]
2. (10 points) This fall the Syracuse football team mostly used 2 quarterbacks: Dungey and Mahoney. Together they threw 318 complete passes. Dungey threw 2 fewer than 3 times the number Mahoney threw.

(a) Write down a system of equations that represent this situation. Use $x$ for the number of complete passes Dungey threw and $y$ for the number that Mahoney threw. Draw a box around your system.

(b) Solve your system. How many did each quarterback throw? Write down intermediate steps and commands.

\[
\begin{align*}
\text{Dungey:} & \quad \text{} \\
\text{Mahoney:} & \quad \text{}
\end{align*}
\]
3. (10 points) A small country's social services agency has two divisions: universal health care (HC) & social protection (SP). In order to produce $1 worth of health care, the agency uses $0.13 in health care and $0.07 in social protection. In order to produce $1 worth of social protection, the agency uses $0.11 in health care and $0.06 in social protection.

(a) Write down the **Input-Output matrix** for this agency. Draw a box around your answer.

(b) At what level should the agency produce to meet a national demand for $632 million in health care and $450 million in social protection. Write down intermediate steps and round to the nearest dollar.

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<th>Health Care</th>
<th>Social Protection</th>
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4. (10 points) Consider the letters O R A N G E S. How many 4-letter words, including nonsense words, can you make from the letters under the following circumstances.

(a) There are no restrictions.

Answer: ________________

(b) The word must start and end with a vowel (O, A or E)

Answer: ________________

5. (8 points) Eighteen people show up at a park to play pick-up volleyball. How many ways can they be divided into 3, 6-person teams: red, white & blue?

Answer: ________________
6. (12 points) Suppose that \( E \& F \) are events with \( \Pr(E) = 0.44 \), \( \Pr(F) = 0.61 \) & \( \Pr(E \cup F) = 0.76 \). Find the following probabilities. Use fractions or decimals rounded to 4 decimal places.

(a) \( \Pr(E \cap F) = \) 

(b) \( \Pr(E \mid F) = \) 

(c) \( \Pr(F \mid E) = \) 

(d) \( \Pr(E^c \mid F) = \)
7. (10 points) Uncle Lumpy plays a game with his nephew Tim using a biased coin. For this coin, \( \Pr(\text{Heads}) = 0.65 \). Tim tosses the coin. If he gets Heads he wins $2, and if he gets Tails he tosses again. On the second toss, if Tim gets Heads he wins $2, and otherwise he has to sing a song.

(a) Draw a tree diagram for this situation and label it with probabilities. Draw a box around your diagram.

(b) What is the probability Tim got Heads on his second toss, given that he won $2? Use fractions or decimals rounded to 4 decimal places.

Answer: ________________
8. (10 points) An urn contains 8 orange balls and 9 black balls. A sample of 3 balls is drawn at random without replacement. Let $X$ be the random variable that denotes the number of orange balls in the sample.

(a) Write down the probability distribution of $X$. Use fractions. Draw a box around your answer.

(b) Find the expected value of $X$.

Answer: ________________

(c) Use words to give an interpretation of the expected value you found in part (b).
9. (10 points) Joe College always forgets to silence his phone during class. In any given class period, his phone has a 16% chance of ringing. Suppose he attends 175 class periods in a semester, and the rings are independent of each other. Write down any calculator commands you use.

(a) **How many classes in the semester do you expect his phone to ring?**

Answer: 

(b) What is the **probability** his phone rings in exactly 25 classes during the semester? Give your answer **rounded to 4 decimal places**.

Answer: 

(c) What is the **probability** his phone rings in more than 30 class periods during the semester? Give your answer **rounded to 4 decimal places**.

Answer: 

10. (10 points) Average sleep times for college freshmen are normally distributed with mean \( \mu = 5.7 \) hours and standard deviation \( \sigma = 1.1 \) hours. For a randomly chosen college freshman Joe College, find the following. Write down any calculator commands you use. Give your answers rounded to 4 decimal places.

(a) The probability Joe gets between 6 and 8 hours of sleep per night.

Answer: 

(b) The probability Joe gets at least 7 hours of sleep per night.

Answer: 

(c) Find the amount of sleep with the property that 40\% of freshmen average less sleep and 60\% average more sleep per night.

Answer: 
11. (10 points) Joe's soccer team is very emotional. If they win a game, there is a 75% chance they will win the next game, but if they lose a game there is only a 30% chance they will win the next game.

(a) Write down the stochastic matrix corresponding to this Markov process. Draw a box around your matrix.

(b) If his team loses a game, what is the probability they will lose two games later? Write down intermediate steps.

Answer: __________

(c) In the long run, what is the probability they will win any particular game? Write your answer as a fraction. Write down intermediate steps.

Answer: __________
12. (10 points) Pet dogs can be classified as Cute Puppy (CP), Housebroken adult (HB), Well-Trained (WT) or Basket Case (BC). Cute Puppies and Housebroken dogs can move between the states, but once a dog is either Well-Trained or a Basket Case their status does not change. The absorbing stochastic matrix for this Markov process is

\[
A = \begin{bmatrix}
1 & 0 & .1 & .3 & & WT \\
0 & 1 & .1 & .1 & & BC \\
0 & 0 & .7 & 0 & & CP \\
0 & 0 & .1 & .6 & & HB \\
\end{bmatrix}
\]

(a) Find the matrix \( F \) and the stable matrix. Write down intermediate steps. Draw boxes around your answers. Give your answers using fractions.

(b) If a dog starts as a Cute Puppy, what is the probability it will eventually be Well-Trained?

Answer: ________________

(d) If a dog starts as a Cute Puppy, about how long will it remain in that state? About how long will it remain a Housebroken adult? Assume that dogs switch states about every 3 months.

Cute Puppy: ________________

Housebroken adult: ________________
13. (10 points) A savings account pays 4.3% interest, compounded weekly. (There are 52 weeks in a year.) Round all answers to two decimal places.

(a) If you deposit $2,000 now, how much will you have in 12 years?

\[ N = \]  
\[ I\% = \]  
\[ PV = \]  
\[ PMT = \]  
\[ FV = \]  
\[ P/Y = \]  
\[ C/Y = \]  

Answer: ________________

(b) How much do you need to deposit today so that you will have $5,000 in 12 years?

\[ N = \]  
\[ I\% = \]  
\[ PV = \]  
\[ PMT = \]  
\[ FV = \]  
\[ P/Y = \]  
\[ C/Y = \]  

Answer: ________________
(c) How much do you need to deposit each week in order to have $5,000 after 12 years?

\[ N = \]
\[ i\% = \]
\[ PV = \]
\[ PMT = \]
\[ FV = \]
\[ P/Y = \]
\[ C/Y = \]

Answer: 

14. (15 points) You take out a 25-year mortgage of $94,000 at 5.4% interest compounded monthly. Round all answers to two decimal places.

(a) How much is your monthly payment?

\[ N = \]
\[ i\% = \]
\[ PV = \]
\[ PMT = \]
\[ FV = \]
\[ P/Y = \]
\[ C/Y = \]

Answer: 

14
(b) At the end of 1 month,

(i) what portion of your first payment goes to interest?

Answer: ________________

(ii) what portion of your first payment goes to principal?

Answer: ________________

(iii) what is your remaining balance after one payment?

Answer: ________________

(b) What is the remaining balance after 16 years?

\[ N = \]

\[ I\% = \]

\[ PV = \]

\[ PMT = \]

\[ FV = \]

\[ P/Y = \]

\[ C/Y = \]

Answer: ________________
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\begin{align*}
3x + 2y - z &= 3 \\
2x - y + 4z &= -1 \\
x + 3y - 5z &= 4
\end{align*}

(b)
\begin{align*}
x - y - z &= 4 \\
2x + 2y + 3z &= -1 \\
4x + z &= 3
\end{align*}

Answer:

Answer:

Answer:
2. (10 points) This fall the Syracuse football team mostly used 2 quarterbacks: Dungey and Mahoney. Together they threw 312 complete passes. Dungey threw 16 fewer than 3 times the number Mahoney threw.

(a) Write down a system of equations that represent this situation. Use \( x \) for the number of complete passes Dungey threw and \( y \) for the number that Mahoney threw. Draw a box around your system.

(b) Solve your system. How many did each quarterback throw? Write down intermediate steps and commands.

Dungey: ______________

Mahoney: ______________
3. (10 points) A small country’s social services agency has two divisions: universal health care (HC) & social protection (SP). In order to produce $1 worth of health care, the agency uses $0.12 in health care and $0.08 in social protection. In order to produce $1 worth of social protection, the agency uses $0.14 in health care and $0.07 in social protection.

(a) Write down the Input-Output matrix for this agency. Draw a box around your answer.

(b) At what level should the agency produce to meet a national demand for $632 million in health care and $450 million in social protection. Write down intermediate steps and round to the nearest dollar.

Health Care: _____________________

Social Protection: ____________________
4. (10 points) Consider the letters O R A N G E. **How many** 4-letter words, including nonsense words, can you make from the letters under the following circumstances.

(a) There are no restrictions.

Answer:_______________________

(b) The word must start and end with a vowel (O, A or E)

Answer:_______________________

5. (8 points) Fifteen people show up at a park to play pick-up basketball. How many ways can they be divided into 3, 5-person teams: red, white & blue?

Answer:_______________________
6. (12 points) Suppose that $E \& F$ are events with $\Pr(E) = 0.38$, $\Pr(F) = 0.57$ & $\Pr(E \cup F) = 0.72$. Find the following probabilities. Use fractions or decimals rounded to 4 decimal places.

(a) $\Pr(E \cap F) =$  

(b) $\Pr(E \mid F) =$  

(c) $\Pr(F \mid E) =$  

(d) $\Pr(E' \mid F) =$  

7. (10 points) Uncle Lumpy plays a game with his nephew Tim using a biased coin. For this coin, \( \Pr(\text{Heads}) = 0.45 \). Tim tosses the coin. If he gets Heads he wins $2, and if he gets Tails he tosses again. On the second toss, if Tim gets Heads he wins $2, and otherwise he has to sing a song.

(a) Draw a tree diagram for this situation and label it with probabilities. Draw a box around your diagram.

(b) What is the probability Tim got Heads on his second toss, given that he won $2? Use fractions or decimals rounded to 4 decimal places.

Answer: ____________________
8. (10 points) An urn contains 8 orange balls and 7 black balls. A sample of 3 balls is drawn at random without replacement. Let $X$ be the random variable that denotes the number of orange balls in the sample.

(a) Write down the probability distribution of $X$. Use fractions. Draw a box around your answer.

(b) Find the expected value of $X$.

Answer: _________________

(c) Use words to give an interpretation of the expected value you found in part (b).
9. (10 points) Joe College always forgets to silence his phone during class. In any given class period, his phone has a 14% chance of ringing. Suppose he attends 150 class periods in a semester, and the rings are independent of each other. Write down any calculator commands you use.

(a) **How many** classes in the semester do you expect his phone to ring?

Answer: ________________

(b) What is the **probability** his phone rings in exactly 25 classes during the semester? Give your answer rounded to 4 decimal places.

Answer: ________________

(c) What is the **probability** his phone rings in more than 20 class periods during the semester? Give your answer rounded to 4 decimal places.

Answer: ________________
10. (10 points) Average sleep times for college freshmen are normally distributed with mean \( \mu = 5.8 \) hours and standard deviation \( \sigma = 1.2 \) hours. For a randomly chosen college freshman Joe College, find the following. Write down any calculator commands you use. Give your answers rounded to 4 decimal places.

(a) The probability Joe gets between 6 and 8 hours of sleep per night.

Answer:______________________

(b) The probability Joe gets at least 7 hours of sleep per night.

Answer:______________________

(c) Find the amount of sleep with the property that 70% of freshmen average less sleep and 30% average more sleep per night.

Answer:______________________
11. (10 points) Joe’s soccer team is very emotional. If they win a game, there is a 70% chance they will win the next game, but if they lose a game there is only a 35% chance they will win the next game.

(a) Write down the stochastic matrix corresponding to this Markov process. Draw a box around your matrix.

(b) If his team loses a game, what is the probability they will lose two games later? Write down intermediate steps.

Answer: 

(c) In the long run, what is the probability they will win any particular game? Write your answer as a fraction. Write down intermediate steps.

Answer:
12. (10 points) Pet dogs can be classified as Cute Puppy (CP), Housebroken adult (HB), Well-Trained (WT) or Basket Case (BC). Cute Puppies and Housebroken dogs can move between the states, but once a dog is either Well-Trained or a Basket Case their status does not change. The absorbing stochastic matrix for this Markov process is

\[
A = \begin{bmatrix}
1 & 0 & .1 & .4 \\
0 & 1 & .1 & .1 \\
0 & 0 & .7 & 0 \\
0 & 0 & .1 & .5
\end{bmatrix}
\]

(a) Find the matrix \( F \) and the stable matrix. Write down intermediate steps. Draw boxes around your answers. Give \( F \) using fractions.

(b) If a dog starts as a Cute Puppy, what is the probability it will eventually be Well-Trained?

Answer:________________________

(d) If a dog starts as a Cute Puppy, about how long will it remain in that state? About how long will it remain a Housebroken adult? Assume that dogs switch states about every 3 months.

Cute Puppy:____________________

Housebroken adult:____________________
13. (10 points) A savings account pays 3.8% interest, compounded weekly. (There are 52 weeks in a year.) Round all answers to two decimal places.

(a) If you deposit $3,000 now, how much will you have in 15 years?

\[ N = \]

\[ i\% = \]

\[ PV = \]

\[ PMT = \]

\[ FV = \]

\[ P/Y = \]

\[ C/Y = \]

Answer: ______________

(b) How much do you need to deposit today so that you will have $8,000 in 15 years?

\[ N = \]

\[ i\% = \]

\[ PV = \]

\[ PMT = \]

\[ FV = \]

\[ P/Y = \]

\[ C/Y = \]

Answer: ______________
(c) How much do you need to deposit each week in order to have $8,000 after 15 years?

\[
N = \\
I\% = \\
PV = \\
PMT = \\
FV = \\
P / Y = \\
C / Y =
\]

Answer: 

14. (15 points) You take out a 20-year mortgage of $98,000 at 5.2% interest compounded monthly. Round all answers to two decimal places.

(a) How much is your monthly payment?

\[
N = \\
I\% = \\
PV = \\
PMT = \\
FV = \\
P / Y = \\
C / Y =
\]

Answer: 

(b) At the end of 1 month,

(i) what portion of your first payment goes to interest?

Answer: ______________________

(ii) what portion of your first payment goes to principal?

Answer: ______________________

(iii) what is your remaining balance after one payment?

Answer: ______________________

(b) What is the remaining balance after 12 years?

\[
\begin{align*}
N &= \\
I\% &= \\
PV &= \\
PMT &= \\
FV &= \\
P/Y &= \\
C/Y &= \\
\end{align*}
\]

Answer: ______________________