May 2nd, 2013

MAT 485 FINAL EXAM

NAME: Student ID:

Instructions: Write your answers and show all your work on this test. To receive credit, you have to show your work! There are 10 questions on 11 pages (including this cover sheet), for a total of 100 points. Be sure to do all of them.

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</table>
1. (10 points) Find the general solution of the differential equation

\[ y' = e^{t-2y}. \]
2. (10 points) Consider the logistic equation model

\[ y' = r \left( 1 - \frac{y}{L} \right) y, \]

where \( r > 0 \) is the intrinsic growth rate and \( L > 0 \) is the carrying capacity. Use the given data values

\[ y(0) = 1, \quad y(1) = \frac{2e}{e + 1}, \quad y(2) = \frac{2e^2}{e^2 + 1}, \]

to solve the parameters \( r \) and \( L \).
3. (10 points) Consider the matrix

\[ A = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} \. 

(a) Solve all the eigenvalues and the related normalized eigenvectors of \( A \).

(b) Using the above eigenvalues and eigenvectors, find the standard decomposition of \( A \), i.e.,

\[ A = VDV^{-1} \. 

4. (10 points) Find the general solution of the differential equation
\[ y'' - 2y' - y = 0. \]
5. (10 points) Find the real general solution of the system of differential equations
\[ \dot{\mathbf{x}} = A\mathbf{x}, \]
where
\[ A = \begin{pmatrix} 1 & -2 \\ 2 & -1 \end{pmatrix}. \]
6. (10 points) Solve the initial valued problem
\[
\begin{align*}
\dot{x} &= Ax + \vec{f} \\
\vec{x}(0) &= \vec{x}_0
\end{align*}
\]
where
\[
A = \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}, \quad \vec{f} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \vec{x}_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.
\]
7. (10 points) Consider the system of differential equations
\[ \dot{x} = Ax + f, \]
where
\[ A = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}, \quad f = \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \]

(a) Find its fixed point.
(b) Check its stability at its fixed point.
8. (10 points) Linearize the system of nonlinear differential equations

\[
\begin{align*}
    x'_1 &= \sin(x_2) \\
    x'_2 &= -\sin(x_1) + x_2
\end{align*}
\]

at the point

\[
    \bar{x}_0 = \begin{pmatrix} \sin \theta \\ \cos \theta \end{pmatrix}
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9. (10 points) Determine the Laplace transform of the function

\[ f(t) = 1 + 3t + e^{-t}\sin(2t). \]
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    & y'' + 3y' + 2y = 0 \\
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\begin{cases}
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\[ f(t) = 1 + 3t + e^{-t} \sin(2t). \]
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 y'' + 3y' + 2y &= 0 \\
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$$\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{f},$$

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(a) Find its fixed point.
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8. (10 points) Linearize the system of nonlinear differential equations
\[
\begin{align*}
    x_1' &= \cos(x_2) \\
    x_2' &= -\cos(x_1) + x_2
\end{align*}
\]
at the point
\[
    \bar{x}_0 = \left( \begin{array}{c}
        \frac{\pi}{2} \\
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