MAT 296 Calculus II
Final Examination
Fall 2014

Print your name: ____________________

Signature: _______________________

SU ID number: ____________
Circle your instructor's name:
Cox    Cuneo    Doerr    Griffin    Heffers    Voldan

Instructions. This examination has 12 problems and 12 pages (including this one).
Make sure your examination is complete before you begin work.

This examination is worth 180 points. The point values are indicated for each of the
problems.

Show your work clearly. All answers must be justified.

The use of or availability of any electronic device, notes or books during this
exam is a violation of the Academic Integrity Policy.

Do NOT write below this line!

1.____  7.____
2.____  8.____
3.____  9.____
4.____  10.____
5.____  11.____
6.____  12.____

TOTAL _______  =  ______%
(1) (a) [8 pts] Set up an integral for the volume of the solid obtained by rotating the region bounded by the curves $y = 2x$ and $y = x^2$ about the $x$-axis. Include a carefully drawn sketch of the region and the solid. DO NOT EVALUATE THE INTEGRAL.

(b) [9 pts] Set up an integral for the volume of the solid obtained by rotating the region bounded by the curves $y = 2x$ and $y = x^2$ about $x = 3$. Include a carefully drawn sketch of the region and the solid. DO NOT EVALUATE THE INTEGRAL.
(2) Evaluate each of the following integrals.

(a) [6 pts] \[\int \sec^5 x \tan x \, dx\]

(b) [10 pts] \[\int x^3 \sqrt{1-x^2} \, dx\]
(3) Evaluate each of the following integrals.

(a) [8 pts] \[ \int x \sin(2x) \, dx \]

(b) [8 pts] \[ \int \frac{3}{x^2 - 7x + 10} \, dx \]
(4) [16 pts] Determine whether the integral is convergent or divergent. If it is convergent, evaluate the integral.

\[ \int_{2}^{5} \frac{1}{\sqrt{5-x}} \, dx \]

\[ \int_{2}^{\infty} \frac{1}{x \ln x} \, dx \]
(5) [15 pts] A conical tank has a height 12 ft and radius at the top of 4 ft. The tank is filled to within 2 feet of the top with oil weighing about 55 lb/ft³.

How much work does it take to pump the oil to a level 3 feet above the rim of the tank?

Include a carefully labeled sketch as part of your solution. DO NOT EVALUATE THE INTEGRAL.
(6) [10 pts] Circle the correct answer for each of the following.

(a) If \( \lim_{n \to \infty} a_n = 0 \), then \( \sum_{n=1}^{\infty} a_n \) is convergent.
   
   TRUE  FALSE

(b) If \( \{a_n\} \) and \( \{b_n\} \) are divergent, then \( \{a_n + b_n\} \) is divergent.
   
   TRUE  FALSE

(c) If \( \sum_{n=1}^{\infty} a_n \) is convergent, then \( \sum_{n=1}^{\infty} |a_n| \) is convergent.
   
   TRUE  FALSE

(d) If \( \sum_{n=1}^{\infty} c_n x^n \) converges when \( x = -4 \), then \( \sum_{n=1}^{\infty} c_n (-3)^n \) converges.
   
   TRUE  FALSE

(e) There exists a power series whose interval of convergence is \( [0, \infty) \).
   
   TRUE  FALSE
(7) [10 pts] Find the values of $x$ for which the series converges. Find the sum of the series for those values of $x$. Justify your reasoning.

\[ \sum_{n=0}^\infty \frac{(x-3)^n}{2^n} \]

(8) [15 pts] Determine whether this series is absolutely convergent, conditionally convergent or divergent. Justify your reasoning.

\[ \sum_{n=1}^\infty (-1)^{n-1} \frac{n}{n^2 + 9} \]
(9) [20 pts] Determine whether each of the following series is absolutely convergent, conditionally convergent or divergent. Justify your reasoning.

\[ \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2 + 1}} \]

\[ \sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n!} \]
(10) [15 pts] Find the radius of convergence and the interval of convergence for the following series. Justify your reasoning.

\[ \sum_{n=0}^{\infty} \frac{(x - 3)^n}{n^3 + 4} \]
(11) [15 pts] Find the Taylor series expansion for $f(x) = \sin x$ centered at $a = \frac{\pi}{2}$. [You may assume that the power series expansion exists.]
(12) [15 pts] Sketch the graph of $r = 1 + \sin(\theta)$ and find the area that it encloses.