Be sure to read the directions of problems carefully, as sometimes you are asked to evaluate an expression and sometimes not. If you can use the result of an earlier problem in a later one, you do not need to redo the earlier problem. To receive full or partial credit the correct work leading to the correct answer must be written down. No books, notes, calculators or other electronic devices, or collaboration with others.

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Instructors: Jaeger, McConnell, Shrestha, Stangle, Ucci, Voldan

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(1) (15 points)

(a) Find the partial fraction decomposition:
\[ \frac{1}{x^2(1+x)} \]

(b) Evaluate the indefinite integral:
\[ \int \cos^2 \theta \, d\theta \]

(c) Find the antiderivative of \((1 - x^2)^{\frac{1}{2}}\) having value 0 at \(x = 0\).
(2) (10 points) Determine if the integral converges or diverges, and find the value if it converges:

(a) \[ \int_0^1 \frac{dx}{x^2(1 + x)} \]

(b) \[ \int_0^\infty xe^{-x} \, dx \]
(3) (15 points) Consider the problem of finding the volume obtained when the region bounded by the curves $y = x - x^2$ and $y = 0$ is revolved around the $x$-axis.

(a) Sketch the curves and shade the region.

(b) Decide on a method (disk/washer or cylindrical shell) and say very briefly why you selected the method you did.

(c) Set up the appropriate integral to find the volume, but do not calculate it.
(4) (10 points) For the curve $y = 2 + 4x^3, 0 \leq x \leq \frac{1}{12}$:

(a) Find the arc length.

(b) Set up, but do not evaluate, the integral needed to find the surface area when the curve is revolved around the y-axis.
(5) (10 points) Determine whether the series shown is convergent or divergent. If it is convergent, find the sum:

$$\sum_{n=1}^{\infty} \frac{2^n}{e^{n-1}}$$
(6) (15 points) Decide whether each of the indicated series converges or diverges. State which test you used and be sure to verify that all the hypotheses of the test are satisfied:

(a) \[ \sum_{n=1}^{\infty} \frac{n}{2n + 1} \]

(b) \[ \sum_{n=3}^{\infty} \frac{1}{n \ln n} \]

(c) \[ \sum_{n=1}^{\infty} \frac{1}{\sqrt[n]{n^3} + 1} \]
(7) (10 points) Starting from the power series representation

\[ \frac{1}{1 - x} = \sum_{n=0}^{\infty} x^n \]

(a) find a power series representation for the function \( f(x) = \frac{x}{1-x^2} \).

(b) find the radius and interval of convergence of the series representing \( f \).
(8) (15 points) Consider the Spiral of Archimedes, whose equation in polar coordinates is

\[ r = a\theta, \quad 0 \leq \theta \leq 2\pi. \]

(Here \( a \) is a positive constant.)

(a) Sketch the curve.

(b) Set up, but do not evaluate, the integral needed to find the arc length.

(c) Set up, but do not evaluate, the integral needed to find the area of the region bounded by the spiral and the positive x-axis, i.e., the region consisting of all points whose polar coordinates satisfy \( 0 \leq r \leq a\theta \) and \( 0 \leq \theta \leq 2\pi \).