There are 10 questions with parts, and the value of each question is marked. There are a possible 120 points on this exam. You have 2 hours.

- Do not open the exam until you are told to do so.
- No notes, calculators, head phones, or electronic devices of any kind are allowed.
- Show all of your work. Answers without supporting work and justification when required, will receive no credit.
- Make sure to answer all of the questions you know how to do. Do not get bogged down on any one problem.
1. (20 points) Calculate the following integrals. Do not simplify your answer.

(a) \( \int_{1}^{e} \left( x^3 + 3x + \frac{1}{x} \right) \, dx = \)

(b) \( \int \frac{1}{x (\ln x)^3} \, dx = \)
(c) \( \int_{0}^{\pi/2} \cos(t) \sin^2(t) \, dt = \)

(d) \( \int \frac{\cos(\sqrt{z} + 3)}{\sqrt{z}} \, dz = \)
2. (10 points) Consider the function

\[ g(x) = \int_0^x (t + 1)^{96} (t - 3)^3 \, dt \]

Find the critical numbers of \( g(x) \). For each critical number, determine whether there is a local minimum, local maximum, or neither, at that critical number.
3. (10 points) Let \( f(x) = x^3 - 12x + 5 \). Find the intervals of increase and decrease, intervals of concavity, inflection points, and local minima and maxima of \( f(x) \).
4. (6 points) Suppose $h(x)$ is a function with $h(5) = 2$, and $h'(x) > 0$ and $h''(x) < 0$ for all $x$.
   (a) Sketch a possible graph of $h(x)$.
   
   (b) Can the equation $h(x) = 0$ have any solutions? If so, how many? If not, why not?

   (c) If $h'(5) = 1/2$, is it possible for $h'(1) = 1/4$? Justify your response.
5. (8 points) A farmer has 24 m of fencing. He wants to enclose a rectangular field bordering a straight river, with no fencing along the river. He also wants an additional fence across the width of the field. See the sketch. Let \( x \) measure the width of the field and \( l \) measure the length; both variables are measured in meters. Find the values of \( x \) and \( l \) that lead to the field with the largest possible area.
6. (8 points) Find the equation of the tangent line to the curve

\[ 3x^2 + 2xy = y^2 \]

at the point \((1, 3)\).
7. (20 points) Calculate the derivatives of the following functions. Do not simplify your answer.

(a) \( f(x) = x \cos x \)

(b) \( g(x) = \sin (e^x) + e^{\cos(x)} \)

(c) \( h(x) = \frac{x^2 - 2}{2x + 1} \)
(d) $i(x) = \int_0^{2x^2} e^{(t^2 + 1)} \, dt$

(e) $j(x) = x^{(x-1)}$
8. (20 points) Calculate the following limits:

(a) \[ \lim_{{x \to \infty}} \left( e^{-x} \right) \sqrt{x} \]

(b) \[ \lim_{{x \to -8^-}} \frac{|x - 8|}{x - 8} \]
(c) \( \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} \)

(d) \( \lim_{t \to \infty} \frac{3t^2 - 7t + 1}{2t^2 - t + 2} \)

(e) \( \lim_{x \to 0} \frac{\sin(4x)}{x} \)
9. (8 points)

(a) State the definition of the derivative of a function $f(x)$ at the point $x = a$.

(b) If a function is continuous, does that mean it is differentiable? If your answer is no, give an example.
10. (10 points) Let $a$ and $b$ be constants and define

$$f(x) = \begin{cases} 
3x^3 - 1, & \text{if } x \leq 0 \\
ax + b, & \text{if } 0 < x \leq 2 \\
x^2 - x - 7, & \text{if } 2 < x
\end{cases}$$

What values of $a$ and $b$ make the function $f(x)$ continuous at all $x$-values?