MAT 286 Final Exam. Max total score 100.

YOUR FULL NAME: ____________________________________________

READ THIS FIRST: Do not open the exam booklet until told to do so.

• You may use a calculator such as TI-83/84, but not a symbolic one such as TI-89. You may not use the textbook, notes, cellphones or other electronic devices.

• Giving or receiving unauthorized aid during an exam is a violation of Syracuse University Academic Integrity Policy.

• Show your work and put a box around your answer.

• Choose one problem between Problem 9 and Problem 10 to solve. Circle the problem which you wish to be graded.

Points:

1. /15
2. /6
3. /10
4. /10
5. /10
6. /15
7. /6
8. /18
9. /10
10. /10

Total: /100
1. (15 pts) Evaluate the following anti-derivatives (indefinite integrals).

a. \[ \int \frac{12x^3 + \sqrt{x}}{x} \, dx \]

b. \[ \int 8x^3 \ln(x) \, dx \]
c. \[ \int \frac{10x}{x^2 + 1} \, dx \]

2. (6 pts) Evaluate the following definite integral.

\[ \int_{0}^{\pi/2} \cos(x) \sin(\sin(x)) \, dx \]
3. (10 pts) Determine whether the following integral converges or diverges. If the integral converges, find its value. If the integral diverges, clearly indicate how you arrived at your answer.

a. \[ \int_{1}^{\infty} \frac{x + 1}{x^2 + 2x} \, dx \]

b. \[ \int_{0}^{\infty} e^{-3x} \, dx \]
4. (10 pts) Evaluate the following double integral

\[ \int_{0}^{2} \int_{0}^{y^2} (3x^2y - y^2) \, dx \, dy \]
5. (10 pts) Solve the following differential equations explicitly for y. If an initial condition is present, find the particular solution, otherwise find the general solution.

a. \( \frac{dy}{dx} = \frac{3y}{x} \); \( y(1) = 15 \) and \( x > 0 \)

b. \( 4 \frac{dy}{dx} - \frac{4y}{x} = 8x, \quad x > 0 \)
6. (15 pts) Suppose we have a function \( y = \sin(x) \).

a. Find the exact area of the region bounded between the function and the \( x \)-axis on the interval \([0, \pi]\).

b. Suppose you revolved the region from part (a) about the \( x \)-axis. Set up an integral that could be used to find the volume of the resulting solid. \textbf{DO NOT EVALUATE}.

c. Find the average value of the function on the interval \([0, \pi]\).
7. (6 pts) Set up integral(s) needed to calculate the area between the curves $y = 5x - x^2$ and $y = x$ on the interval $[0, 7]$. DO NOT EVALUATE THE INTEGRAL(S). It may be helpful to draw a picture.

8. (18 pts) Suppose that the weight of a substance changes at a rate of $r(t) = 5e^{0.2t}$, where $t$ is the time. (Here, $r(t)$ is measured using pounds per hour and $t$ is in hours.)

a. If $w(t)$ denotes the weight of the substance at time $t$, what units are appropriate for the function $w(t)$?

b. What is the total change in the weight of the object between $t = 5$ hours and $t = 10$ hours? Include appropriate units in your answer.
c. If we also know that the weight of the substance is initially 5 pounds, write down a formula for the weight function $w(t)$. 
Choose one of the following two problems to do. Circle which problem you would like you have graded.

Problem 9  OR  Problem 10

9. (10pts) Suppose a town has a population of 25,000 people. An epidemic in this community starts when 40 people get infected, and 8 days later, 320 people are infected. Assume that the rate at which the epidemic spreads is proportional to the number of people infected and the number of people not yet infected.

(a) Find an equation for the number of people infected at time $t$ ($t$ in days).

(b) How many people are infected 25 days into the epidemic?
Consider a tank containing 100 gallons of pure water. A saltwater solution with 1.8 pounds of salt per gallon is flowing into the tank at a rate of 3 gallons per minute and the solution flows out at the same rate. Let $y(t)$ denote the amount of salt (measured in pounds) in the tank at time $t$ (measured in minutes).

Find an equation for $y(t)$ that gives the amount of salt in the tank at a given time $t$. 