MAT 286

December 11, 2013

Final Exam

Show your work. No work-no credit!

Name ____________________________ SU ID # ____________________________

Please circle your instructor’s NAME.

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1. Find the following integrals. [5 points for each problem.]

(a) \( \int (4x^3 + 6x - 5) \, dx \)

(b) \( \int 4e^{-3x} \, dx \)
(c) \int \frac{x}{x^2 + 5} \, dx

(d) \int x \sin x \, dx
2. For (a) and (b), find the values of the definite integrals. For (c) and (d), each of the improper integrals, determine if it converges or diverges, and find the value of each that converges. [5 points for each problem.]

(a) \( \int_{x=1}^{x=8} (x^3)dx \)

(b) \( \int_{x=(\frac{\pi}{6})}^{x=(\frac{\pi}{2})} \cos x \, dx \)
(c) \[ \int_{0}^{\infty} \frac{x}{x^2 + 5} \, dx \]

(d) \[ \int_{-\infty}^{\infty} \frac{2x}{(x^2 + 1)^2} \, dx \]
3. (a) Find the average value of the function, \( f(x) = \sqrt{x + 1} \), over the interval \([3, 8]\).[5 points]

(b) Find the volume under the given surface,

\[ f(x, y) = (1 - x^2) y, \]

and above the rectangle with the given boundaries, \(-1 \leq x \leq 1, 0 \leq y \leq 4\).[5 points]
4. (a) **Evaluate** the following integral [10 points]

\[
\int_{y=4}^{y=5} x \sqrt{x^2 + 3y} \, dy.
\]

(b) **Evaluate** the following double integral. [10 points]

\[
\int_{y=1}^{y=5} \int_{x=1}^{x=y} (4x - 3y^2) \, dxdy
\]
5. Solve the following differential equations. (Solve the initial value problems if the initial values are given, otherwise find the general solutions.) [5 points for each problem.]

(a) \( \frac{dy}{dx} = \frac{2x}{3y^2} \).

(b) \( \frac{dy}{dx} = \frac{1}{y}, \quad y = -3, \text{ when } x = 0. \)
(c) \( \frac{dy}{dx} + 3y = e^{-2x} \).

(d) \( x \frac{dy}{dx} + 5y = x^3 \), for \( x > 0 \); with the initial condition \( y(1) = 2 \).
6. Use *Euler's method* to approximate the indicated *function value* to three decimal places, using $h = 0.1$.

\[ \frac{dy}{dx} = x + \sqrt{y} ; \quad y(0) = 1 ; \quad \text{find} \quad y(0.4) . \]

[10 points]