Circle your instructor's name:
Yi Wong

Show your work. You may use a calculator, but you must write the steps you take with the calculator. A correct answer with no work will receive no credit. Point values are indicated by the problem.

1. Suppose that the slope of the tangent line to a function $f(x)$ at any $x$ is given by

$$f'(x) = x^2 - e^x + 1.$$

If the graph $y = f(x)$ goes through the point $(0, 1)$, find the function $f(x)$.

1. ______________________

(10 points)
3. Evaluate the following indefinite integrals. Circle your answers.

a) 
\[ \int \sin x \cos x \, dx \]
(10 points)

b) 
\[ \int \frac{1}{x (\ln x)^3} \, dx \]
(10 points)
3c)

$$\int xe^{-x} \, dx$$

(10 points)
4. Determine whether the following improper integrals converge or diverge. If the integral converges, give the value of the convergent integral.

a) \[ \int_{0}^{\infty} \frac{x}{\sqrt{x^2 + 1}} \, dx. \]

(5 points)

b) \[ \int_{1}^{\infty} x^{-\frac{1}{4}} \, dx. \]

(5 points)
5. Find the general solution of the differential equation

\[ \frac{dy}{dx} = \frac{y}{x^2}, \quad y > 0. \]

(10 points)
6. Evaluate the double integral of the function $f(x, y) = x^2 + xy$ over the region $R$, where $R = \{(x, y) : 0 \leq x \leq 1, x^2 \leq y \leq x\}$. Give the exact answer (no decimals).

(10 points)
7. Solve the following differential equation subject to the initial condition that $y = 2$ when $x = 1$.

$$x \frac{dy}{dx} - 3y + 2 = 0.$$
8. An influenza epidemic spreads at a rate proportional to the product of the number of people infected and the number of people not yet infected with the constant of proportionality $k = 0.01$. Assume that 10 people are infected at the beginning of the epidemic in a community of 100 people.

a) Find the function $y(t)$ that gives the number of people infected after $t$ days.

(10 points)

$$\frac{1}{y(100 - y)} = \frac{1}{100} \left( \frac{1}{y} + \frac{1}{100 - y} \right).$$

8a) 

b) After how many days (to the nearest 0.1 day) will half the community be infected?

(5 points)

8b) 

1. Suppose that the slope of the tangent line to a function \( f(x) \) at any \( x \) is given by

\[
f'(x) = x^2 - e^x + 1.
\]

If the graph \( y = f(x) \) goes through the point \( (0, 1) \), find the function \( f(x) \).

1. __________________________

(10 points)
2. Consider the following function \( f(x) \) on the interval \([0, 1]\).

\[
f(x) = 3x^2 + \sin(2\pi x).
\]

a) Use calculus to find the **exact area** bounded by the curve and the \( x \)-axis on the given interval.

(Show all steps including the appropriate antiderivative and the evaluations.
No credit will be given for providing only the final numerical value.)

2a. ____________________________

(10 points)

b) Set up, but do not evaluate the integral that gives the volume of the solid formed by revolving the region bounded by the curve \( f(x) = 3x^2 + \sin(2\pi x) \) and the \( x \)-axis on the interval \([0, 1]\) about the \( x \)-axis. Circle your integral.

(5 points)
3. Evaluate the following indefinite integrals. Circle your answers.

a) \[ \int \sin x \cos x \, dx \]

(10 points)

b) \[ \int \frac{1}{x (\ln x)^3} \, dx \]

(10 points)
3c) \[ \int xe^{-x} \, dx \]

(10 points)
4. Determine whether the following improper integrals converge or diverge. If the integral converges, give the value of the convergent integral.

a)

\[ \int_0^\infty \frac{x}{\sqrt{x^2 + 1}} \, dx. \]

(5 points)

b)

\[ \int_1^\infty x^{-\frac{3}{2}} \, dx. \]

(5 points)
5. Find the general solution of the differential equation

\[
\frac{dy}{dx} = \frac{y}{x^2}, \quad y > 0.
\]

(10 points)
6. Evaluate the double integral of the function $f(x, y) = x^2 + xy$ over the region $R$, where $R = \{(x, y) : 0 \leq x \leq 1, x^2 \leq y \leq x\}$. Give the exact answer (no decimals).

6. ________________________

(10 points)
7. Solve the following differential equation subject to the initial condition that $y = 2$ when $x = 1$.

$$x \frac{dy}{dx} - 3y + 2 = 0.$$

(10 points)
8. An influenza epidemic spreads at a rate proportional to the product of the number of people infected and the number of people not yet infected. Assume that 100 people are infected at the beginning of the epidemic in a community of 10,000 people, and 500 are infected 10 days later.

a) Find the function $y(t)$ that gives the number of people infected after $t$ days.

(10 points)

Note that \[
\frac{1}{y(10000 - y)} = \frac{1}{10000} \left( \frac{1}{y} + \frac{1}{10000 - y} \right).
\]

b) When will half the community be infected?

(5 points)