Final Exam
MAT 285 Spring 2013

Circle your instructor's name:
Biermann  Mkhatshwa  Walpole  Boyd  Yaziki

Turn off all cell phones and similar electronic devices.

Show your work and justify your answers. You may use a TI 83/84 Plus or other approved calculator, but you must write the steps you take with the calculator. A correct answer with no work will not receive credit. Point values are indicated.

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1. (10 points) The growth of a bacterial colony is modeled by the function

\[ A(t) = 2770e^{0.05t}, \]

where \( A \) is the population \( t \) days after the initial measurement.

a) Find the population 42 days after the initial measurement. **Round your answer to the nearest whole number.**

1a. ________________

b) After how many days does the population first reach 90,000? **Round your answer to two decimal places.**

1b. ________________
2. (10 points) Find the following limits. Circle your answers.

a) \[ \lim_{x \to 3} \frac{x^2 + 3x - 18}{x^2 - 9x + 18} \]

b) \[ \lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} \]

c) \[ \lim_{x \to \infty} \frac{-x^3 - 2x + 1}{2x^3 - 3x^2} \]
3. (10 points) Look at the graph of the function below. Find the indicated limits and function values. Write “doesn’t exist” if the value or limit does not exist.

a) \( \lim_{x \to 1^-} f(x) = \) ____________

b) \( f(1) = \) ____________

c) \( f(-3) = \) ____________

d) \( \lim_{x \to -3^-} f(x) = \) ____________

e) \( \lim_{x \to -3^+} f(x) = \) ____________
4. (10 points) Use the definition of the derivative to find $f'(x)$ for

$$f(x) = x^2 - 3x.$$

You must use the definition of the derivative to receive any credit for this problem. Circle your answer.
5. (35 points) Find the derivative of each function. Use appropriate notation. Circle your answer. Do not simplify.

(i) \( f(x) = x^2 \tan(5x) \)

(ii) \( g(t) = e^{3t}t \)

(iii) \( y = \sqrt{x} + 3^{-x} \)

(iv) \( f(x) = \frac{x^6 + 4}{\cos x} \)

(v) \( g(x) = \ln (4 - 3x^2) \)
6. (15 points) A spherical balloon is being inflated at a constant rate of 15 cm$^3$ per second. How fast is the radius of the balloon increasing when radius is 9 cm? **Round your answer to two decimal places.**
7. (15 points) Consider the function \( f(x) = x^3 - 6x^2 - 36x + 7 \). Make sure that your answers are easy to find.

a) Identify the intervals on which the function is increasing and on which the function is decreasing.

b) Identify the intervals on which the function is concave up and on which the function is concave down.

c) Identify any relative extremes. Indicate if an extreme is a maximum or minimum.
8. (15 points) A worker needs to build a rectangular pen with two dividers against a long, straight wall as shown in the diagram. The worker has 660 feet of fence. What are the dimensions of the outside of the pen that has the maximum area?

\[ x \]
9. (20 points) a) Use implicit differentiation to find \( \frac{dy}{dx} \) for
\[ x^3y + 2x^4 - 2y^3 = -1. \]

9a. 

b) Find the equation of the line tangent to the curve in part a) at the point \((-1, 1)\).

9b. 

10. (10 points) Find $f_y(x, y)$ and $f_{xx}(x, y)$ for

$$f(x, y) = \sin(5x^2y) - 4xy^2 + 6x.$$ 

$$f_y(x, y) = \phantom{\text{expression}}$$

$$f_{xx}(x, y) = \phantom{\text{expression}}$$
11. (15 points) Find and classify each critical point (as relative maximum, relative minimum, or saddle point) of

\[ f(x, y) = y^3 + 3x^2y - 6x^2 - 6y^2 - 5. \]

Circle your answers.
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1. (10 points) The growth of a bacterial colony is modeled by the function

\[ A(t) = 3350e^{0.07t}, \]

where \( A \) is the population \( t \) days after the initial measurement.

a) Find the population 30 days after the initial measurement. Round your answer to the nearest whole number.

1a. 

b) After how many days does the population first reach 150,000? Round your answer to two decimal places.

1b. 

2. (10 points) Find the following limits. Circle your answers.

a) 
\[ \lim_{x \to 1} \frac{x^2 + 3x - 4}{x^2 - 5x + 4} \]

b) 
\[ \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} \]

c) 
\[ \lim_{x \to \infty} \frac{x^4 - 2x + 1}{-2x^5 + 3x^4 + 5} \]
3. (10 points) Look at the graph of the function below. Find the indicated limits and function values. Write “doesn’t exist” if the value or limit does not exist.

a) \( \lim_{x \to 2} f(x) = \) 

b) \( f(2) = \) 

c) \( f(-2) = \) 

d) \( \lim_{x \to -2^-} f(x) = \) 

e) \( \lim_{x \to -2^+} f(x) = \)
4. (10 points) Use the definition of the derivative to find $f'(x)$ for

$$f(x) = x^2 + 5x.$$ 

You *must* use the definition of the derivative to receive any credit for this problem. Circle your answer.
5. (35 points) Find the derivative of each function. Use appropriate notation. Circle your answer. Do not simplify.

(i) \( f(x) = x^3 \cos(6x) \)

(ii) \( g(t) = e^{-2t+t^2} \)

(iii) \( y = 2^x - \frac{1}{\sqrt{x}} \)

(iv) \( f(x) = \frac{x^5 - x}{\tan x} \)

(v) \( g(x) = \ln (5 - x^3) \)
6. (15 points) A spherical balloon is being inflated at a constant rate of 10 cm³ per second. How fast is the radius of the balloon increasing when radius is 5 cm? **Round your answer to two decimal places.**
7. (15 points) Consider the function \( f(x) = x^3 - 6x^2 - 36x + 7 \). Make sure that your answers are easy to find.

   a) Identify the intervals on which the function is increasing and on which the function is decreasing.

   b) Identify the intervals on which the function is concave up and on which the function is concave down.

   c) Identify any relative extremes. Indicate if an extreme is a maximum or minimum.
9. (20 points) a) Use implicit differentiation to find \( \frac{dy}{dx} \) for

\[ x^2y + 3x^2 + 2y = 6. \]

b) Find the equation of the line tangent to the curve in part a) at the point \((-1,1)\).
10. (10 points) Find $f_x(x, y)$ and $f_{yy}(x, y)$ for

$$f(x, y) = \cos(3xy^2) - 4x^2y + 6y.$$ 

$$f_x(x, y) = \underline{\hphantom{8.96} \; \underline{\hphantom{8.96}}}$$

$$f_{yy}(x, y) = \underline{\hphantom{8.96} \; \underline{\hphantom{8.96}}}$$
11. (15 points) Find and classify each critical point (as relative maximum, relative minimum, or saddle point) of

\[ f(x, y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2. \]

Circle your answers.