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• You may use a calculator. You may not share a calculator with another student.

• Show all work required to solve the problems. Answers without supporting work and justification when required, will receive no credit.

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• Make sure to answer all of the questions that you know how to answer. Don’t get bogged down on any one problem.

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1. (10 pts) The supply and demand equations for a particular product are

\[ p = 50 + .08q \quad \quad p = 130 - .12q, \]

respectively, where \( p \) represents the price per unit in dollars and \( q \) represents the number of units sold per time period.

(a) Find the equilibrium price \( p \) and quantity \( q \).

(b) Find the equilibrium price when a tax of 5 cents (\$0.05) per unit is imposed on the supplier.
2. (10 pts) (a) Let \( a = \ln 2, \ b = \ln 3, \) and \( c = \ln 5. \) Write \[
\ln \left( \frac{18}{5} \right)
\]
in terms of \( a, \ b, \) and \( c. \)

(b) Write \[
5 \ln x - \ln(x^3 + x^2) + \frac{1}{2} \ln(x^4 + 1)
\]
as a single logarithm.
3. (10 pts) A manufacturer will produce 1400 units when the price is $10 per unit and 1600 units when the price is $14 per unit.

(a) Find the linear equation relating $p$, the price per unit, and $q$, the number of units produced.

(b) How many units will be produced if the price is $17.50?
4. (7 pts) Find an equation of the line tangent to the graph of

\[ f(x) = (4x - 3)^5 + 8 \]

at \( x = 1 \).

5. (8 pts) Evaluate the following limits. Show your reasoning.

(a) \( \lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 + 3x - 10} \)

(b) \( \lim_{x \to \infty} \frac{5x^4 + 3x^2 + 7}{x^2 + 3x^3 - 4x^4} \)
6. (20 pts) Find the derivative, \( f'(x) \), of each function. You do not need to simplify your answer.

(a) \( f(x) = 2x^3 e^{5x} \)

(b) \( f(x) = \frac{5x^2 - 7x}{x^3 + 6x + 7} \)

(c) \( f(x) = \sqrt{x} + \frac{3}{\sqrt{x}} + \frac{1}{x^3} \)

(d) \( f(x) = (x^{2/3} + \ln x)^7 \)

(e) \( f(x) = \ln(4x^3 + 3x^2) \)
7. (10 pts) The demand function for the product manufactured by a monopolist is

\[ p = 22 - 0.16q. \]

The average cost of producing this product is \( \bar{c} = 0.04q + 10 + \frac{40}{q} \) dollars per unit.

(a) Express the monopolist’s profit as a function of the number \( q \) of items sold.

(b) Use calculus to determine at what value of output \( q \) profit will be maximized? What is the price when the profit is maximized? What is the maximum profit?
8. (10 pts) Consider the function \( f(x) = 2x^3 - 3x^2 - 36x + 22 \). Use calculus to answer the following questions and show your work!

(a) Find the critical values of \( f(x) \).

(b) Find all the open intervals on which \( f(x) \) is increasing and all the open intervals on which \( f(x) \) is decreasing.

(c) Determine the \( x \)-coordinates of all relative maxima and all relative minima of \( f(x) \).
(d) Find the open intervals on which the graph of $f(x)$ is concave up and the open intervals on which the graph of $f(x)$ is concave down.

(e) Find the $x$-coordinate of all inflection points on the graph of $f(x)$. If there aren't any, write NONE.

9. (7 pts) The demand function for a certain product is $p = 300 - q^2$, where $q$ is the quantity of the product produced and sold while $p$ is the unit price when $q$ units are produced.

(a) Find the point elasticity of demand when $q = 12$.

(b) Is demand elastic, inelastic, or unit elastic when $q = 12$?
10. (8 pts) (a) Evaluate the following indefinite integral.

\[
\int 3\sqrt{x} + \frac{1}{4x} + \frac{4}{x^3} - 3e^x \, dx.
\]

(b) The marginal revenue function for a certain product is \( \frac{dr}{dq} = 100 + 60q - 9q^2 \). Find the demand function for the product.
MAT 284  
Final Exam B  
May 4, 2015

Name: _________________________________

SU ID Number: __________________________

Signature: ______________________________

Instructor: John McDermott (CIRCLE ONE)

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1. (10 pts) The supply and demand equations for a particular product are

\[ p = 60 + .06q \quad \quad p = 150 - .14q, \]

respectively, where \( p \) represents the price per unit in dollars and \( q \) represents the number of units sold per time period.

(a) Find the equilibrium price \( p \) and quantity \( q \).

(b) Find the equilibrium price when a tax of 5 cents (\$0.05) per unit is imposed on the supplier.
2. (10 pts) (a) Let \( a = \ln 2, b = \ln 3, \) and \( c = \ln 5. \) Write 
\[
\ln \left( \frac{50}{3} \right)
\]
in terms of \( a, b, \) and \( c. \)

(b) Write \( 4 \ln x - \ln(x^4 + 1) + \frac{1}{2} \ln(x^3 + x^2) \) as a single logarithm.
3. (10 pts) A manufacturer will produce 1200 units when the price is $10 per unit and 1400 units when the price is $14 per unit.

(a) Find the linear equation relating $p$, the price per unit, and $q$, the number of units produced.

(b) How many units will be produced if the price is $18.50?
4. (7 pts) Find an equation of the line tangent to the graph of

\[ f(x) = (3x - 2)^4 + 6 \]

at \( x = 1 \).

5. (8 pts) Evaluate the following limits. Show your reasoning.

(a) \( \lim_{x \to 2} \frac{x^2 - 7x + 10}{x^2 + 2x - 8} \)

(b) \( \lim_{x \to \infty} \frac{x^2 + 3x^3 - 4x^4}{5x^4 + 3x^2 + 7} \)
6. (20 pts) Find the derivative, \( f'(x) \), of each function. You do not need to simplify your answer.

(a) \( f(x) = 5x^4 e^{3x} \)

(b) \( f(x) = \frac{x^3 + 6x + 7}{5x^2 - 7x} \)

(c) \( f(x) = \sqrt{x} + \frac{5}{\sqrt{x}} + \frac{1}{x^4} \)

(d) \( f(x) = (x^{3/5} + \ln x)^6 \)

(e) \( f(x) = \ln(3x^4 + 5x^3) \)
7. (10 pts) The demand function for the product manufactured by a monopolist is

\[ p = 24 - 0.14q. \]

The average cost of producing this product is \( \bar{c} = 0.06q + 14 + \frac{40}{q} \) dollars per unit.

(a) Express the monopolist's profit as a function of the number \( q \) of items sold.

(b) Use calculus to determine at what value of output \( q \) profit will be maximized? What is the price when the profit is maximized? What is the maximum profit?
8. (10 pts) Consider the function \( f(x) = 2x^3 + 3x^2 - 36x + 22 \). Use calculus to answer the following questions and show your work!

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(e) Find the $x$-coordinate of all inflection points on the graph of $f(x)$. If there aren’t any, write NONE.

9. (7 pts) The demand function for a certain product is $p = 400 - q^2$, where $q$ is the quantity of the product produced and sold while $p$ is the unit price when $q$ units are produced.

(a) Find the point elasticity of demand when $q = 12$.

(b) Is demand elastic, inelastic, or unit elastic when $q = 12$?
10. (8 pts) (a) Evaluate the following indefinite integral.

\[ \int 3\sqrt{x} + \frac{1}{4x} + \frac{5}{x^4} - 6e^x \, dx. \]

(b) The marginal revenue function for a certain product is \( \frac{dr}{dq} = 120 + 50q - 12q^2 \). Find the demand function for the product.