MAT 183 Final Exam, Ver. Q1 Fall 2012

Signature:

Instructions: Write the answers and show the main steps of your work on this test sheet.

There are 17 questions on 16 pages (including this cover).

Be sure you have all 16 pages (8 sheets) and that they are all of the same version.

You should spend no more than 8 minutes on each problem; be sure that you get to the easier parts of each problem.

The Final Exam is scored on a basis of 100 points and will count 25% of your final grade.

You must show your work to get full credit! If you use the calculator, write down your input.

DO NOT WRITE ON THE REST OF THIS COVER SHEET!

Problem 1 (6)  Problem 6 (4)  Problem 12 (5)
Problem 2 (6)  Problem 7 (6)  Problem 13 (6)
Problem 3 (6)  Problem 8 (6)  Problem 14 (6)
Problem 4 (9)  Problem 9 (6)  Problem 15 (6)
Problem 5 (7)  Problem 10 (6)  Problem 16 (6)
Problem 11 (5)
Test 1 Total (34)  Test 2 Total (33)  Test 3 Total (33)

EXAM TOTAL (100)
Problem 1. (6 points) In each case below, a system of three equations in $w, x, y$ and $z$ (in that order) has been put in row-reduced echelon form. You are to interpret this matrix. In each case circle one of “No Solution”, “Unique Solution” or “Many Solutions.” If you circle “Unique Solution”, fill in the values of $w, x, y$ and $z$. If you circle “Many Solutions”, fill in the values for 3 of the variables in terms of the 4th.

(i) (2 points)

\[
\begin{bmatrix}
1 & 0 & 0 & 4 \\
0 & 1 & -1 & -3 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

NoSolution $w =$

\[
\begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & -1 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

ManySolutions $x =$

\[
\begin{bmatrix}
1 & 0 & 0 & 4 \\
0 & 1 & 0 & -3 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

UniqueSolution $y =$

\[
\begin{bmatrix}
0 & 0 & 0 & 1
\end{bmatrix}
\]

UniqueSolution $z =$

(ii) (2 points)

\[
\begin{bmatrix}
1 & 0 & 0 & 4 \\
0 & 1 & 0 & -3 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

NoSolution $w =$

\[
\begin{bmatrix}
1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

ManySolutions $x =$

\[
\begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

UniqueSolution $y =$

\[
\begin{bmatrix}
0 & 0 & 0 & 1
\end{bmatrix}
\]

UniqueSolution $z =$
Problem 2. (6 points) Katie is buying fish for her aquarium. Goldfish cost $2 each, angelfish are $4 each, and catfish are $6 dollars each. Katie wants to have twice as many goldfish as angelfish and catfish combined. She wants to spend a total of $140 and buy 45 fish all together. How many of each kind of fish should she buy?

(i) (2 points) Identify the variables. Use g, a and c and explain in words EXACTLY what each variable represents:

\[ g, \]
\[ a, \]
\[ c, \]

(ii) (4 points) Give the system of equations that you plan to use. Do NOT Solve this system!
Problem 3. (6 points)

(i) (3 points) A corporation has an electronics division and an industrial equipment division. For each $1 worth of output, the electronics division needs $0.06 worth of electronics and $0.10 worth of equipment. For each $1 worth of output, the industrial equipment division needs $0.02 worth of electronics and $0.05 worth of equipment. The current demand is for $1,630,000 worth of electronics and $2,500,000 worth of industrial equipment. Set up the input-output matrix and demand matrix. Do NOT solve.

(ii) (3 points) A simplified economy has the input-output matrix, $A$, and the demand matrix, $D$, given below. Find at what level the divisions need to produce to meet demand.

$$A = \begin{bmatrix} 0.05 & 0.08 \\ 0.02 & 0.06 \end{bmatrix} \quad D = \begin{bmatrix} 928,000 \\ 440,300 \end{bmatrix}$$
Problem 4. You want to make a 5 letter word out of the letters F, G, H, I and J.

(i) (3 points) How many different 5 letter words are possible if repetitions are allowed?

(ii) (3 points) How many different 5 letter words are possible if repetitions are not allowed?

(iii) (3 points) How many different 5 letter words begin with a H if repetitions are not allowed?
Problem 5. (7 points) A survey of 100 college freshmen who exercise regularly found that 41 jog, 30 swim, 25 cycle, 6 jog and swim, 4 jog and cycle, 5 swim and cycle, and 1 does all three.

(i) (2 points) In the following Venn Diagram, shade the region that represents those students that Swim, but do not jog or cycle.

(ii) (5 points) Compute the number of students who do not jog, swim or cycle. Show your work by either giving the formula that you use or by filling in the number of students in each region of the following Venn Diagram.
Problem 6. (4 points) In a certain class 35% of the voters are republicans, 31% democrats, 22% independent and the remaining voters belong to the green party. A random voter is selected.

(i) What is the probability that he or she is not a democrat?

(ii) What is the probability that he or she is a member of the green party?

Problem 7. (6 points) Consider events $X$, $Y$ and $Z$ with

- $\Pr(X) = \frac{1}{2}$, $\Pr(Y) = \frac{1}{3}$, $\Pr(Z) = \frac{1}{4}$, $\Pr(Y \text{ and } Z) = \frac{1}{6}$

and events $X$ and $Y$ are independent.

(i) What is the probability that the both events $X$ and $Y$ occur?

(ii) What is the probability that either event $Y$ or $Z$ or both occur?

(iii) What is the probability that event $Y$ occurs given that event $Z$ occurred?
Problem 8. (6 points) You have a box containing 4 white markers and 1 black marker; you also have a white urn containing 4 orange markers and 3 blue markers and a black urn containing 2 orange markers and 5 blue markers.

You select one marker from the box. If it is white, select a marker from the white urn; if it is black, select a marker from the black urn.

(i) (5 points) Label the tree diagram of this experiment. Be sure to include labels for all edges and for the end of each path!

(ii) (1 point) Circle the most likely color for the second marker you choose:

orange blue
Problem 9. (6 points) Consider the following probability distribution for the random variable $X$

<table>
<thead>
<tr>
<th>$k$</th>
<th>$P(X = k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>0.4</td>
</tr>
</tbody>
</table>

(i) (3 points) Compute the expected value of $X$ (Show your work or your calculator entry)

$$E(X) = _____$$

(ii) (3 points) Compute the standard deviation of $X$ (Show your work or your calculator entry)

$$\sigma(X) = _____$$
Problem 10. (6 points) A factory produces lightbulbs, and 6% of them are defective.

(i) (4 points) Given a random sample of 50 lightbulbs, find the mean and standard deviation for the number of defective lightbulbs in the sample.

\[ \mu = \underline{\hspace{1cm}} \]

\[ \sigma = \underline{\hspace{1cm}} \]

(ii) (2 points) Given a random sample of 50 lightbulbs, what is the probability that exactly 4 of them are defective?

Problem 11. (5 points) The scores on a certain test are normally distributed with mean 73 and standard deviation 10 as pictured below. What percentage of the students scored between 65 and 85?

YOU MUST SHOW YOUR WORK!

[If you use the calculator, write out exactly what you enter; if you use the table show your calculations.]
Problem 12. (5 points) Consider the following absorbing stochastic matrix:

\[
A = \begin{bmatrix}
  1 & 0 & 0.2 & 0 \\
  0 & 1 & 0.4 & 0.1 \\
  0 & 0 & 0.3 & 0.4 \\
  0 & 0 & 0.1 & 0.5 \\
\end{bmatrix}
\]

(i) (3 points) Identify the absorbing states of A.

(ii) (2 points) Determine the fundamental matrix for A.
Problem 13. (6 points) For a certain group of states, it was observed that 60% of Democratic governors were succeeded by Democrats and 40% by Republicans. Also, 45% of the Republican governors were succeeded by Democrats and 55% by Republicans.

(i) (3 points) Write a stochastic matrix describing the transitions in this problem.

(ii) (3 points) Suppose 35% of the current governors are Democrats and 65% are Republicans. Assuming that the current trend holds, what percent of the governors will be Democrats after the next election?
Problem 14. (6 points) Consider the following investments at 6.2% APR, compounded monthly. To get credit you must show how you computed your answer. If you use the TVM Solver, fill in the table in the left hand margin otherwise write out the formula that you use.

(i) (2 points) If you invest $20,000 today, what will this investment be worth at the end of 8 years?

<table>
<thead>
<tr>
<th>N=</th>
<th>I%=</th>
<th>PV=</th>
<th>PMT=</th>
<th>FV=</th>
<th>P/Y=</th>
<th>C/Y=</th>
</tr>
</thead>
</table>

(ii) (2 points) Approximately how many months would it take for an investment of $20,000 to grow to $30,000?

<table>
<thead>
<tr>
<th>N=</th>
<th>I%=</th>
<th>PV=</th>
<th>PMT=</th>
<th>FV=</th>
<th>P/Y=</th>
<th>C/Y=</th>
</tr>
</thead>
</table>

(iii) (2 points) How much should you invest today if you want to have $20,000 in 5 years?

<table>
<thead>
<tr>
<th>N=</th>
<th>I%=</th>
<th>PV=</th>
<th>PMT=</th>
<th>FV=</th>
<th>P/Y=</th>
<th>C/Y=</th>
</tr>
</thead>
</table>
Problem 15. (6 points) Consider a savings account paying 6.2% interest compounded monthly into which you make regular deposits at the end of each month. To get credit you must show how you computed your answer. If you use the TVM Solver fill in the table in the left hand margin otherwise write out the formula that you use.

(i) (2 points) If your payments are $1,540 each, how much will be in your account at the end of 5 years?

(ii) (2 points) How much of this is interest that you have earned?
Circle one:

$8,672.67 $10,741.70 $13,948.55 $15,599.93 NONE OF THESE

(iii) (2 points) How much should you deposit today if you want to withdraw $1,200 at the end of each month for the next 5 years?
Problem 16. (6 points) Consider a $220,000, 25 year mortgage at 6.2% interest compounded monthly. To get credit you must show how you computed your answer.

(i) (2 points) Compute your monthly payments:

N= \\
I%= \\
PV= \\
PMT= \\
FV= \\
P/Y= \\
C/Y= \\

(ii) (2 points) How much of your first payment is interest? You must show your computations!

(iii) (2 points) If the bank charges you 1 point, how much of the loan actually goes toward the purchase of the house? Circle one:

$222,200  $220,000  $217800  $215600  NONE OF THESE
Problem 17. (4 points)

(i) You borrow $18,000 at 8% for two years with payments computed by the add-on method. Your monthly payments are? Circle one:

- $580.00
- $676.67
- $773.33
- $870.00
- NONE OF THESE

(ii) You are planning to set up an IRA for your retirement. You expect to be in the 26% tax bracket until you retire and you expect to be in the 29% bracket throughout your retirement. What type of IRA should you set up? Circle one:

- TRADITIONAL IRA
- ROTH IRA
- IT MAKES NO DIFFERENCE
MAT 183 Final Exam, Ver. Q₂ Fall 2012

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Problem 11 (5)  Problem 17 (4)

Test 1 Total (34)  Test 2 Total (33)  Test 3 Total (33)

EXAM TOTAL (100)
Problem 18. (6 points) In each case below, a system of three equations in $w$, $x$, $y$, and $z$ (in that order) has been put in row-reduced echelon form. You are to interpret this matrix. In each case circle one of "No Solution", "Unique Solution" or "Many Solutions." If you circle "Unique Solution", fill in the values of $w$, $x$, $y$, and $z$. If you circle "Many Solutions", fill in the values for 3 of the variables in terms of the 4th.

(i) (2 points)

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 7 \\
0 & 1 & -1 & 0 & -4 \\
0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

NoSolution

\[
w = \\
x = 1 \\
y = \\
z =
\]

(ii) (2 points)

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 7 \\
0 & 1 & 0 & 0 & -4 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
\]

NoSolution

\[
w = \\
x = \\
y = 1 \\
z =
\]

(iii) (2 points)

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 7 \\
0 & 1 & 0 & 0 & -4 \\
0 & 0 & 1 & -1 & 1 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

NoSolution

\[
w = \\
x = \\
y = 1 \\
z =
\]
Problem 19. (6 points) Katie is buying fish for her aquarium. Goldfish cost $2 each, angelfish are $4 each, and catfish are $6 dollars each. Katie wants to have twice as many goldfish as angelfish and catfish combined. She wants to spend a total of $120 and buy 45 fish all together. How many of each kind of fish should she buy?

(i) (2 points) Identify the variables. Use $g$, $a$ and $c$ and explain in words EXACTLY what each variable represents:

$$g,$$
$$a,$$
$$c,$$

(ii) (4 points) Give the system of equations that you plan to use. Do NOT Solve this system!
Problem 20. (6 points)

(i) (3 points) A corporation has an electronics division and an industrial equipment division. For each $1 worth of output, the electronics division needs $0.06 worth of electronics and $0.10 worth of equipment. For each $1 worth of output, the industrial equipment division needs $0.02 worth of electronics and $0.05 worth of equipment. The current demand is for $1,630,000 worth of electronics and $2,500,000 worth of industrial equipment. Set up the input-output matrix and demand matrix. Do NOT solve.

(ii) (3 points) A simplified economy has the input-output matrix, $A$, and the demand matrix, $D$, given below. Find at what level the divisions need to produce to meet demand.

\[
A = \begin{bmatrix} 0.05 & 0.08 \\ 0.02 & 0.06 \end{bmatrix}, \quad D = \begin{bmatrix} 938,000 \\ 420,300 \end{bmatrix}
\]
Problem 21. You want to make a 5 letter word out of the letters K, L, M, N and O.

(i) (3 points) How many different 5 letter words are possible if repetitions are allowed?

(ii) (3 points) How many different 5 letter words are possible if repetitions are not allowed?

(iii) (3 points) How many different 5 letter words begin with a N if repetitions are not allowed?
Problem 22. (7 points) A survey of 100 college freshmen who exercise regularly found that 41 jog, 30 swim, 25 cycle, 6 jog and swim, 4 jog and cycle, 5 swim and cycle, and 1 does all three.

(i) (2 points) In the following Venn Diagram, shade the region that represents those students that jog, but do not swim or cycle.

(ii) (5 points) Compute the number of students who do not jog, swim or cycle. Show your work by either giving the formula that you use or by filling in the number of students in each region of the following Venn Diagram.
Problem 23. (4 points) In a certain class 32% of the voters are republicans, 31% democrats, 22% independent and the remaining voters belong to the green party. A random voter is selected.

(i) What is the probability that he or she is not a democrat?

(ii) What is the probability that he or she is a member of the green party?

Problem 24. (6 points) Consider events $X$, $Y$ and $Z$ with

- $\Pr(X) = \frac{1}{4}$, $\Pr(Y) = \frac{1}{2}$, $\Pr(Z) = \frac{1}{3}$, $\Pr(Y \text{ and } Z) = \frac{1}{6}$

and events $X$ and $Y$ are independent.

(i) What is the probability that the both events $X$ and $Y$ occur?

(ii) What is the probability that either event $Y$ or $Z$ or both occur?

(iii) What is the probability that event $Y$ occurs given that event $Z$ occurred?
Problem 25. (6 points) You have a box containing 4 white markers and 2 black markers; you also have a white urn containing 4 orange markers and 3 blue markers and a black urn containing 2 orange markers and 5 blue markers.

You select one marker from the box. If it is white, select a marker from the white urn; if it is black, select a marker from the black urn.

(i) (5 points) Label the tree diagram of this experiment. Be sure to include labels for all edges and for the end of each path!

(ii) (1 point) Circle the most likely color for the second marker you choose:

orange  blue
Problem 26. (6 points) Consider the following probability distribution for the random variable $X$

<table>
<thead>
<tr>
<th>$k$</th>
<th>$P(X = k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

(i) (3 points) Compute the expected value of $X$ (Show your work or your calculator entry)

\[ E(X) = \] 

(ii) (3 points) Compute the standard deviation of $X$ (Show your work or your calculator entry)

\[ \sigma(X) = \]
Problem 27. (6 points) A factory produces lightbulbs, and 6% of them are defective.

(i) (4 points) Given a random sample of 50 lightbulbs, find the mean and standard deviation for the number of defective lightbulbs in the sample.

\[ \mu = \ldots \]

\[ \sigma = \ldots \]

(ii) (2 points) Given a random sample of 50 lightbulbs, what is the probability that exactly 4 of them are defective?

Problem 28. (5 points) The scores on a certain test are normally distributed with mean 73 and standard deviation 10 as pictured below. What percentage of the students scored between 60 and 85? YOU MUST SHOW YOUR WORK!

[If you use the calculator, write out exactly what you enter; if you use the table show your calculations.]
Problem 29. (5 points) Consider the following absorbing stochastic matrix:

\[
A = \begin{bmatrix}
1 & 0 & 0.2 & 0 \\
0 & 1 & 0.4 & 0.1 \\
0 & 0 & 0.3 & 0.4 \\
0 & 0 & 0.1 & 0.5 \\
\end{bmatrix}
\]

(i) (3 points) Identify the absorbing states of A.

(ii) (2 points) Determine the fundamental matrix for A.
Problem 30. (6 points) For a certain group of states, it was observed that 60% of Democratic governors were succeeded by Democrats and 40% by Republicans. Also, 45% of the Republican governors were succeeded by Democrats and 55% by Republicans.

(i) (3 points) Write a stochastic matrix describing the transitions in this problem.

(ii) (3 points) Suppose 35% of the current governors are Democrats and 65% are Republicans. Assuming that the current trend holds, what percent of the governors will be Democrats after the next election?
Problem 31. (6 points) Consider the following investments at 4.8% APR, compounded monthly. To get credit you must show how you computed your answer. If you use the TVM Solver, fill in the table in the left hand margin otherwise write out the formula that you use.

(i) (2 points) If you invest $20,000 today, what will this investment be worth at the end of 8 years?

<table>
<thead>
<tr>
<th>N=</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I%=</td>
<td></td>
</tr>
<tr>
<td>PV=</td>
<td></td>
</tr>
<tr>
<td>PMT=</td>
<td></td>
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<tr>
<td>FV=</td>
<td></td>
</tr>
<tr>
<td>P/Y=</td>
<td></td>
</tr>
<tr>
<td>C/Y=</td>
<td></td>
</tr>
</tbody>
</table>

(ii) (2 points) Approximately how many months would it take for an investment of $20,000 to grow to $30,000?

<table>
<thead>
<tr>
<th>N=</th>
<th></th>
</tr>
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<tr>
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<tr>
<td>P/Y=</td>
<td></td>
</tr>
<tr>
<td>C/Y=</td>
<td></td>
</tr>
</tbody>
</table>

(iii) (2 points) How much should you invest today if you want to have $20,000 in 5 years?

<table>
<thead>
<tr>
<th>N=</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I%=</td>
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<tr>
<td>FV=</td>
<td></td>
</tr>
<tr>
<td>P/Y=</td>
<td></td>
</tr>
<tr>
<td>C/Y=</td>
<td></td>
</tr>
</tbody>
</table>
Problem 32. (6 points) Consider a savings account paying 4.8% interest compounded monthly into which you make regular deposits at the end of each month. To get credit you must show how you computed your answer. If you use the TVM Solver fill in the table in the left hand margin otherwise write out the formula that you use.

(i) (2 points) If your payments are $1,540 each, how much will be in your account at the end of 5 years?

(ii) (2 points) How much of this is interest that you have earned? Circle one:

- $8,672.67
- $10,741.70
- $13,948.55
- $15,599.93
- NONE OF THESE

(iii) (2 points) How much should you deposit today if you want to withdraw $1,200 at the end of each month for the next 5 years?
Problem 33. (6 points) Consider a $220,000, 25 year mortgage at 4.8% interest compounded monthly. To get credit you must show how you computed your answer.

(i) (2 points) Compute your monthly payments:

\[ \begin{array}{l}
N= \\
I\%= \\
PV= \\
PMT= \\
FV= \\
P/Y= \\
C/Y= \\
\end{array} \]

(ii) (2 points) How much of your first payment is interest? You must show your computations!

(iii) (2 points) If the bank charges you 1 point, how much of the loan actually goes toward the purchase of the house? Circle one:

$222,200  \quad$220,000  \quad$217800  \quad$215600  \quad$200000  \quad$150000  \quad$100000  \quad$50000  \quad NONE OF THESE
Problem 34. (4 points)

(i) You borrow $18,000 at 8% for two years with payments computed by the add-on method. Your monthly payments are? Circle one:

- $580.00
- $616.61
- $773.33
- $810.00
- NONE OF THESE

(ii) You are planning to set up an IRA for your retirement. You expect to be in the 26% tax bracket until you retire and you expect to be in the 29% bracket throughout your retirement. What type of IRA should you set up? Circle one:

- TRADITIONAL IRA
- ROTH IRA
- IT MAKES NO DIFFERENCE
MAT 183 Final Exam, Ver. Q3 Fall 2012

Signature:

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Test 1 Total (34) Test 2 Total (33) Test 3 Total (33)

EXAM TOTAL (100)
Problem 35. (6 points) In each case below, a system of three equations in \( w, x, y \) and \( z \) (in that order) has been put in row-reduced echelon form. You are to interpret this matrix. In each case circle one of “No Solution”, “Unique Solution” or “Many Solutions.” If you circle “Unique Solution”, fill in the values of \( w, x, y \) and \( z \). If you circle “Many Solutions”, fill in the values for 3 of the variables in terms of the 4th.

(i) (2 points)
\[
\begin{bmatrix}
1 & 0 & 0 & -4 \\
0 & 1 & 0 & 5 \\
0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1
\end{bmatrix}
\]
- NoSolution
- ManySolutions
- UniqueSolution

(ii) (2 points)
\[
\begin{bmatrix}
1 & 0 & 0 & -4 \\
0 & 1 & -1 & 5 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
- NoSolution
- ManySolutions
- UniqueSolution

(iii) (2 points)
\[
\begin{bmatrix}
1 & 0 & 0 & -4 \\
0 & 1 & 0 & 5 \\
0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
- NoSolution
- ManySolutions
- UniqueSolution

\[
w =
\]
\[
x =
\]
\[
y =
\]
\[
z =
\]
Problem 36. (6 points) Katie is buying fish for her aquarium. Goldfish cost $2 each, angelfish are $4 each, and catfish are $6 dollars each. Katie wants to have twice as many goldfish as angelfish and catfish combined. She wants to spend a total of $160 and buy 45 fish all together. How many of each kind of fish should she buy?

(i) (2 points) Identify the variables. Use g, a and c and explain in words EXACTLY what each variable represents:

- g,
- a,
- c,

(ii) (4 points) Give the system of equations that you plan to use. Do NOT Solve this system!
Problem 37. (6 points)

(i) (3 points) A corporation has an electronics division and an industrial equipment division. For each \$1\ worth of output, the electronics division needs \$0.06\ worth of electronics and \$0.10\ worth of equipment. For each \$1\ worth of output, the industrial equipment division needs \$0.02\ worth of electronics and \$0.05\ worth of equipment. The current demand is for \$1,630,000\ worth of electronics and \$2,500,000\ worth of industrial equipment. Set up the input-output matrix and demand matrix. Do NOT solve.

(ii) (3 points) A simplified economy has the input-output matrix, \( A \), and the demand matrix, \( D \), given below. Find at what level the divisions need to produce to meet demand.

\[
A = \begin{bmatrix} 0.05 & 0.08 \\ 0.02 & 0.06 \end{bmatrix} \quad D = \begin{bmatrix} 948,000 \\ 420,300 \end{bmatrix}
\]
Problem 38. You want to make a 5 letter word out of the letters P, Q, R, S and T.
   (i) (3 points) How many different 5 letter words are possible if repetitions are allowed?

   (ii) (3 points) How many different 5 letter words are possible if repetitions are not allowed?

   (iii) (3 points) How many different 5 letter words begin with a T if repetitions are not allowed?
Problem 39. (7 points) A survey of 100 college freshmen who exercise regularly found that 41 jog, 30 swim, 25 cycle, 6 jog and swim, 4 jog and cycle, 5 swim and cycle, and 1 does all three.

(i) (2 points) In the following Venn Diagram, shade the region that represents those students that cycle, but do not swim or jog.

(ii) (5 points) Compute the number of students who do not jog, swim or cycle. Show your work by either giving the formula that you use or by filling in the number of students in each region of the following Venn Diagram.
Problem 40. (4 points) In a certain class 32% of the voters are republicans, 35% democrats, 22% independent and the remaining voters belong to the green party. A random voter is selected.

(i) What is the probability that he or she is not a democrat?

(ii) What is the probability that he or she is a member of the green party?

Problem 41. (6 points) Consider events $X$, $Y$ and $Z$ with $\Pr(X)=\frac{1}{3}$, $\Pr(Y)=\frac{1}{2}$, $\Pr(Z)=\frac{1}{2}$, $\Pr(Y \text{ and } Z)=\frac{1}{6}$ and events $X$ and $Y$ are independent.

(i) What is the probability that the both events $X$ and $Y$ occur?

(ii) What is the probability that either event $Y$ or $Z$ or both occur?

(iii) What is the probability that event $Y$ occurs given that event $Z$ occurred?
Problem 42. (6 points) You have a box containing 5 white markers and 1 black marker; you also have a white urn containing 4 orange markers and 3 blue markers and a black urn containing 2 orange markers and 5 blue markers.

You select one marker from the box. If it is white, select a marker from the white urn; if it is black, select a marker from the black urn.

(i) (5 points) Label the tree diagram of this experiment. Be sure to include labels for all edges and for the end of each path!

(ii) (1 point) Circle the most likely color for the second marker you choose:

orange blue
Problem 43. (6 points) Consider the following probability distribution for the random variable $X$

<table>
<thead>
<tr>
<th>$k$</th>
<th>$P(X = k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>0.5</td>
</tr>
</tbody>
</table>

(i) (3 points) Compute the expected value of $X$ (Show your work or your calculator entry)

$E(X) = ____$

(ii) (3 points) Compute the standard deviation of $X$ (Show your work or your calculator entry)

$\sigma(X) = ____$
Problem 44. (6 points) A factory produces lightbulbs, and 6% of them are defective.

(i) (4 points) Given a random sample of 50 lightbulbs, find the mean and standard deviation for the number of defective lightbulbs in the sample.

\[ \mu = \underline{____} \]

\[ \sigma = \underline{____} \]

(ii) (2 points) Given a random sample of 50 lightbulbs, what is the probability that exactly 4 of them are defective?

Problem 45. (5 points) The scores on a certain test are normally distributed with mean 73 and standard deviation 10 as pictured below. What percentage of the students scored between 65 and 90?

YOU MUST SHOW YOUR WORK!

[If you use the calculator, write out exactly what you enter; if you use the table show your calculations.]
Problem 46. (5 points) Consider the following absorbing stochastic matrix:

\[
A = \begin{bmatrix}
1 & 0 & 0.2 & 0 \\
0 & 1 & 0.4 & 0.1 \\
0 & 0 & 0.3 & 0.4 \\
0 & 0 & 0.1 & 0.5 \\
\end{bmatrix}
\]

(i) (3 points) Identify the absorbing states of A.

(ii) (2 points) Determine the fundamental matrix for A.
Problem 47. (6 points) For a certain group of states, it was observed that 60% of Democratic governors were succeeded by Democrats and 40% by Republicans. Also, 45% of the Republican governors were succeeded by Democrats and 55% by Republicans.

(i) (3 points) Write a stochastic matrix describing the transitions in this problem.

(ii) (3 points) Suppose 35% of the current governors are Democrats and 65% are Republicans. Assuming that the current trend holds, what percent of the governors will be Democrats after the next election?
Problem 48. (6 points) Consider the following investments at 5.3% APR, compounded monthly. To get credit you must show how you computed your answer. If you use the TVM Solver, fill in the table in the left hand margin otherwise write out the formula that you use.

(i) (2 points) If you invest $20,000 today, what will this investment be worth at the end of 8 years?

<table>
<thead>
<tr>
<th>N=</th>
<th>I%=</th>
<th>PV=</th>
<th>PMT=</th>
<th>FV=</th>
<th>P/Y=</th>
<th>C/Y=</th>
</tr>
</thead>
</table>

(ii) (2 points) Approximately how many months would it take for an investment of $20,000 to grow to $30,000?

<table>
<thead>
<tr>
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<th>I%=</th>
<th>PV=</th>
<th>PMT=</th>
<th>FV=</th>
<th>P/Y=</th>
<th>C/Y=</th>
</tr>
</thead>
</table>

(iii) (2 points) How much should you invest today if you want to have $20,000 in 5 years?

<table>
<thead>
<tr>
<th>N=</th>
<th>I%=</th>
<th>PV=</th>
<th>PMT=</th>
<th>FV=</th>
<th>P/Y=</th>
<th>C/Y=</th>
</tr>
</thead>
</table>
Problem 49. (6 points) Consider a savings account paying 5.3% interest compounded monthly into which you make regular deposits at the end of each month. To get credit you must show how you computed your answer. If you use the TVM Solver fill in the table in the left hand margin otherwise write out the formula that you use.

(i) (2 points) If your payments are $1,540 each, how much will be in your account at the end of 5 years?

(ii) (2 points) How much of this is interest that you have earned?

Circle one:

- $8,672.67
- $10,741.70
- $13,948.55
- $15,599.93
- NONE OF THESE

(iii) (2 points) How much should you deposit today if you want to withdraw $1,200 at the end of each month for the next 5 years?
Problem 50. (6 points) Consider a $220,000, 25 year mortgage at 5.3% interest compounded monthly. To get credit you must show how you computed your answer.

(i) (2 points) Compute your monthly payments:

\[
\begin{align*}
N &= \\
I\% &= \\
PV &= \\
PMT &= \\
FV &= \\
P/Y &= \\
C/Y &= \\
\end{align*}
\]

(ii) (2 points) How much of your first payment is interest? You must show your computations!

(iii) (2 points) If the bank charges you 1 point, how much of the loan actually goes toward the purchase of the house? Circle one:

\[
\begin{align*}
$222,200 & \quad $220,000 & \quad $217,800 & \quad $215,600 & \quad \text{NONE OF THESE} \\
\end{align*}
\]
Problem 51. (4 points)

(i) You borrow $18,000 at 8% for two years with payments computed by the add-on method. Your monthly payments are? Circle one:

$580.00  $676.67  $773.33  $870.00  NONE OF THESE

(ii) You are planning to set up an IRA for your retirement. You expect to be in the 26% tax bracket until you retire and you expect to be in the 29% bracket throughout your retirement. What type of IRA should you set up? Circle one:

TRADITIONAL IRA  ROTH IRA  IT MAKES NO DIFFERENCE
Probability and Statistical formulas:

(i) \( Pr(E') = 1 - Pr(E) \)

(ii) \( Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F) \)

(iii) \( Pr(E \cap F) = Pr(E) \times Pr(F) \) if and only if \( E \) and \( F \) are independent

(iv) \( Pr(E \cap F) = 0 \) if and only if \( E \) and \( F \) are mutually exclusive

(v) \( Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)} \)

(vi) \( \mu = x_1p_1 + \cdots + x_np_n \)

(vii) \( \sigma = \sqrt{(x_1-\mu)^2p_1 + \cdots + (x_n-\mu)^2p_n} = \sqrt{x_1^2p_1 + \cdots + x_n^2p_n - \mu^2} \)

(viii) For the binomial random variable \( X \) with \( N \) trials and probability \( p \) of success:

\[
Pr(X = k) = \binom{N}{k}p^k(1-p)^{N-k}
\]

\( \mu_X = Np \) and \( \sigma_X = \sqrt{Np(1-p)} \)

(ix) \( Z = \frac{X-\mu}{\sigma} \)

Stochastic Formulas

Stable Distribution for regular matrices: \( X = \begin{bmatrix} x \\ y \end{bmatrix} \) satisfies:

\[
\text{sum of the entries of } X = 1
\]

\[
AX = X
\]

Standard Form for absorbing matrices: \( A = \begin{bmatrix} I & S \\ 0 & R \end{bmatrix} \)

Stable Matrix for absorbing matrices: \( \begin{bmatrix} I & S(I-R)^{-1} \\ 0 & 0 \end{bmatrix} \)

Finance formulas

Simple interest: \( F = (1 + yr)P \).

Effective interest: \( 1 + e = (1 + \frac{r}{c})^c \),

where \( c \) is the number of compounding periods per year.

Periodic rate: \( i = \frac{r}{c} \),

where \( c \) is the number of compounding periods per year.

Compound interest: \( B_k = (1+i)B_{k-1} \) giving \( F = (1+i)^nP \)

Increasing annuity: \( B_k = (1+i)B_{k-1} + R \) giving \( F = \frac{(1+i)^n-1}{i}R \)

Decreasing annuity: \( B_k = (1+i)B_{k-1} - R \) giving \( P = \frac{(1+i)^n-1}{i(1+i)n}R \)
**TABLE 1 Areas under the standard normal curve**

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</table>
MAT 183 Final Exam, Ver. Q4 Fall 2012

Signature:

Instructions: Write the answers and show the main steps of your work on this test sheet.

There are 17 questions on 16 pages (including this cover).

Be sure you have all 16 pages (8 sheets) and that they are all of the same version.

You should spend no more than 8 minutes on each problem; be sure that you get to the easier parts of each problem.

The Final Exam is scored on a basis of 100 points and will count 25% of your final grade.

You must show your work to get full credit! If you use the calculator, write down your input.

DO NOT WRITE ON THE REST OF THIS COVER SHEET!

| Problem 1 (6) | Problem 6 (4) | Problem 12 (5) |
| Problem 2 (6) | Problem 7 (6) | Problem 13 (6) |
| Problem 3 (6) | Problem 8 (6) | Problem 14 (6) |
| Problem 4 (9) | Problem 9 (6) | Problem 15 (6) |
| Problem 5 (7) | Problem 10 (6) | Problem 16 (6) |
| Problem 11 (5) | Test 1 Total (34) | Test 2 Total (33) |

Test 3 Total (33)

EXAM TOTAL (100)
Problem 52. (6 points) In each case below, a system of three equations in \( w, x, y \) and \( z \) (in that order) has been put in row-reduced echelon form. You are to interpret this matrix. In each case circle one of “No Solution”, “Unique Solution” or “Many Solutions.” If you circle “Unique Solution”, fill in the values of \( w, x, y \) and \( z \). If you circle “Many Solutions”, fill in the values for 3 of the variables in terms of the 4th.

(i) (2 points) 
\[
\begin{bmatrix}
1 & 0 & 0 & 0 & -2 \\
0 & 1 & 0 & 0 & 8 \\
0 & 0 & 1 & -1 & 1 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

NoSolution \( x = \) 
ManySolutions \( y = \) 
UniqueSolution \( z = \)

(ii) (2 points) 
\[
\begin{bmatrix}
1 & 0 & 0 & 0 & -2 \\
0 & 1 & 0 & 0 & 8 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
\]

NoSolution \( x = \) 
ManySolutions \( y = \) 
UniqueSolution \( z = \)

(iii) (2 points) 
\[
\begin{bmatrix}
1 & 0 & 0 & 0 & -2 \\
0 & 1 & -1 & 0 & 8 \\
0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

NoSolution \( x = \) 
ManySolutions \( y = \) 
UniqueSolution \( z = \)
Problem 53. (6 points) Katie is buying fish for her aquarium. Goldfish cost $2 each, angelfish are $4 each, and catfish are $6 dollars each. Katie wants to have twice as many goldfish as angelfish and catfish combined. She wants to spend a total of $130 and buy 45 fish all together. How many of each kind of fish should she buy?

(i) (2 points) Identify the variables. Use g, a and c and explain in words EXACTLY what each variable represents:

\[ g, \]
\[ a, \]
\[ c. \]

(ii) (4 points) Give the system of equations that you plan to use. Do NOT Solve this system!
Problem 54. (6 points)

(i) (3 points) A corporation has an electronics division and an industrial equipment division. For each $1 worth of output, the electronics division needs $0.06 worth of electronics and $0.10 worth of equipment. For each $1 worth of output, the industrial equipment division needs $0.02 worth of electronics and $0.05 worth of equipment. The current demand is for $1,630,000 worth of electronics and $2,500,000 worth of industrial equipment. Set up the input-output matrix and demand matrix. Do NOT solve.

(ii) (3 points) A simplified economy has the input-output matrix, $A$, and the demand matrix, $D$, given below. Find at what level the divisions need to produce to meet demand.

\[
A = \begin{bmatrix}
0.05 & 0.08 \\
0.02 & 0.06 \\
\end{bmatrix} \quad D = \begin{bmatrix}
918,000 \\
450,300 \\
\end{bmatrix}
\]
Problem 55. You want to make a 5 letter word out of the letters A, B, C, D and E.

(i) (3 points) How many different 5 letter words are possible if repetitions are allowed?

(ii) (3 points) How many different 5 letter words are possible if repetitions are not allowed?

(iii) (3 points) How many different 5 letter words begin with a B if repetitions are not allowed?
Problem 56. (7 points) A survey of 100 college freshmen who exercise regularly found that 41 jog, 30 swim, 25 cycle, 6 jog and swim, 4 jog and cycle, 5 swim and cycle, and 1 does all three.

(i) (2 points) In the following Venn Diagram, shade the region that represents those students that jog, but do not swim or cycle.

(ii) (5 points) Compute the number of students who do not jog, swim or cycle. Show your work by either giving the formula that you use or by filling in the number of students in each region of the following Venn Diagram.
Problem 57. (4 points) In a certain class 32% of the voters are republicans, 31% democrats, 25% independent and the remaining voters belong to the green party. A random voter is selected.

(i) What is the probability that he or she is not a democrat?

(ii) What is the probability that he or she is a member of the green party?

Problem 58. (6 points) Consider events $X$, $Y$ and $Z$ with
\[ \Pr(X) = \frac{1}{3}, \quad \Pr(Y) = \frac{1}{2}, \quad \Pr(Z) = \frac{1}{4}, \quad \Pr(Y \text{ and } Z) = \frac{1}{6} \]
and events $X$ and $Y$ are independent.

(i) What is the probability that the both events $X$ and $Y$ occur?

(ii) What is the probability that either event $Y$ or $Z$ or both occur?

(iii) What is the probability that event $Y$ occurs given that event $Z$ occurred?
Problem 59. (6 points) You have a box containing 5 white markers and 2 black marker; you also have a white urn containing 4 orange markers and 3 blue markers and a black urn containing 2 orange markers and 5 blue markers.

You select one marker from the box. If it is white, select a marker from the white urn; if it is black, select a marker from the black urn.

(i) (5 points) Label the tree diagram of this experiment. Be sure to include labels for all edges and for the end of each path!

(ii) (1 point) Circle the most likely color for the second marker you choose:

orange   blue
Problem 60. (6 points) Consider the following probability distribution for the random variable $X$

<table>
<thead>
<tr>
<th>$k$</th>
<th>$P(X = k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0.4</td>
</tr>
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</table>

(i) (3 points) Compute the expected value of $X$ (Show your work or your calculator entry)

$E(X) = \ldots$

(ii) (3 points) Compute the standard deviation of $X$ (Show your work or your calculator entry)

$\sigma(X) = \ldots$
Problem 61. (6 points) A factory produces lightbulbs, and 6% of them are defective.

(i) (4 points) Given a random sample of 50 lightbulbs, find the mean and standard deviation for the number of defective lightbulbs in the sample.

\[ \mu = \ldots \]

\[ \sigma = \ldots \]

(ii) (2 points) Given a random sample of 50 lightbulbs, what is the probability that exactly 4 of them are defective?

Problem 62. (5 points) The scores on a certain test are normally distributed with mean 73 and standard deviation 10 as pictured below. What percentage of the students scored between 60 and 90? 

You must show your work!

[If you use the calculator, write out exactly what you enter; if you use the table show your calculations.]
Problem 63. (5 points) Consider the following absorbing stochastic matrix:

\[
A = \begin{bmatrix}
1 & 0 & 0.2 & 0 \\
0 & 1 & 0.4 & 0.1 \\
0 & 0 & 0.3 & 0.4 \\
0 & 0 & 0.1 & 0.5 \\
\end{bmatrix}
\]

(i) (3 points) Identify the absorbing states of \( A \).

(ii) (2 points) Determine the fundamental matrix for \( A \).
Problem 64. (6 points) For a certain group of states, it was observed that 60% of Democratic governors were succeeded by Democrats and 40% by Republicans. Also, 45% of the Republican governors were succeeded by Democrats and 55% by Republicans.

(i) (3 points) Write a stochastic matrix describing the transitions in this problem.

(ii) (3 points) Suppose 35% of the current governors are Democrats and 65% are Republicans. Assuming that the current trend holds, what percent of the governors will be Democrats after the next election?
Problem 65. (6 points) Consider the following investments at 5.9% APR, compounded monthly. To get credit you must show how you computed your answer. If you use the TVM Solver, fill in the table in the left hand margin otherwise write out the formula that you use.

(i) (2 points) If you invest $20,000 today, what will this investment be worth at the end of 8 years?

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(ii) (2 points) Approximately how many months would it take for an investment of $20,000 to grow to $30,000?

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</table>

(iii) (2 points) How much should you invest today if you want to have $20,000 in 5 years?

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<td>C/Y=</td>
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</table>
Problem 66. (6 points) Consider a savings account paying 5.9% interest compounded monthly into which you make regular deposits at the end of each month. To get credit you must show how you computed your answer. If you use the TVM Solver fill in the table in the left hand margin otherwise write out the formula that you use.

(i) (2 points) If your payments are $1,540 each, how much will be in your account at the end of 5 years?

(ii) (2 points) How much of this is interest that you have earned?
Circle one:
$8,672.67  $10,741.70  $13,948.55  $15,599.93  NONE OF THESE

(iii) (2 points) How much should you deposit today if you want to withdraw $1,200 at the end of each month for the next 5 years?
Problem 67. (6 points) Consider a $220,000, 25 year mortgage at 5.9% interest compounded monthly. To get credit you must show how you computed your answer.

(i) (2 points) Compute your monthly payments:

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<td>P/Y=</td>
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<td>C/Y=</td>
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</table>

(ii) (2 points) How much of your first payment is interest? You must show your computations!

(iii) (2 points) If the bank charges you 1 point, how much of the loan actually goes toward the purchase of the house? Circle one:

$222,200 $220,000 $217800 $215600 NONE OF THESE
Problem 68. (4 points)

(i) You borrow $18,000 at 8% for two years with payments computed by the add-on method. Your monthly payments are? Circle one:

$580.00  $676.67  $773.33  $870.00  NONE OF THESE

(ii) You are planning to set up an IRA for your retirement. You expect to be in the 26% tax bracket until you retire and you expect to be in the 29% bracket throughout your retirement. What type of IRA should you set up? Circle one:

TRADITIONAL IRA  ROTH IRA  IT MAKES NO DIFFERENCE
1. Answer the following question parts.
   a) Solve the following system of linear equations.

   \[
   \begin{align*}
   x + 2y + 4z &= -2 \\
   y + 5z &= 2 \\
   -2x - 4y - 3z &= 9
   \end{align*}
   \]

   b) A system of linear equations has already been put into a reduced-row echelon form matrix. Write down the solution if possible. If no solution is possible, write “No Solution” and justify your answer.

   \[
   \begin{bmatrix}
   1 & 0 & 0 & 0 \\
   0 & 1 & 0 & 0 \\
   0 & 0 & 0 & 1
   \end{bmatrix}
   \]
2. It is found that the number of married and single adults in a certain town are subject to the following statistics. Suppose that $x$ and $y$ denote the number of married and single adults, respectively, in a given year (say as of January 1) and let $m$, $s$, denote the corresponding numbers for the following year. Then

$$0.8x + 0.3y = m$$
$$0.2x + 0.7y = s$$

a) Write down the matrix equation, $AX = B$.

b) Solve the matrix equation for $X = \begin{bmatrix} x \\ y \end{bmatrix}$.

c) Suppose that in a given year there were found to be 100,000 married adults and 50,000 single adults. How many married (respectively, single) adults were there the preceding (earlier) year?
3. Given \( n(S) = 5 \), \( n(T) = 4 \), \( n(S') = 5 \), and \( n(S \cap T) = 2 \). Find the following:

a) Find \( n(U) \).

b) Find \( n((S' \cup T')') \).

c) Find \( n(S \cup T) \).
4. Answer the following question parts.

   a) Twenty athletes enter an Olympic event. How many different possibilities are there for winning the Gold Medal, Silver Medal, and Bronze Medal.

   b) Suppose that you own 10 sweaters and are going on a trip. How many ways can you select six of them to leave at home?
5. Ten percent of all undergraduates at a university are chemistry majors. Eight students are selected at random.

a) Find the probability that exactly 2 are chemistry majors.

b) Find the probability that less than 2 are chemistry majors.
6. Let \( E \) and \( F \) be events with \( \Pr(E) = .3 \), \( \Pr(F) = .6 \), and \( \Pr(E \cup F) = .7 \). Find the following:

a) \( \Pr(E \cap F) \).

b) \( \Pr(E|F) \).

c) Are \( E \) and \( F \) independent events? **Justify.**
7. Given the following probability distribution table, compute the mean \( (\mu_x) \), variance \( (\sigma_x^2) \), and standard deviation \( (\sigma_x) \).

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Probability</th>
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<tbody>
<tr>
<td>5</td>
<td>2/46</td>
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<tr>
<td>6</td>
<td>2/46</td>
</tr>
<tr>
<td>7</td>
<td>12/46</td>
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<tr>
<td>8</td>
<td>20/46</td>
</tr>
<tr>
<td>9</td>
<td>10/46</td>
</tr>
</tbody>
</table>
8. Answer the following question parts.

   a) A fair coin is tossed seven times. What is the probability of obtaining four heads and three tails?

   b) An urn contains six white balls and five red balls. A sample of four balls is selected at random from the urn. What is the probability that the sample contains two white balls and two red balls?
9. Two thousand dollars is deposited in a savings account at 4% interest compounded semiannually.

   a) Find the balance after 7 years.

   \[
   \begin{align*}
   N &= \quad \text{N} \\
   I\% &= \quad \text{I\%} \\
   PV &= \quad \text{PV} \\
   PMT &= \quad \text{PMT} \\
   FV &= \quad \text{FV} \\
   P/Y &= \quad \text{P/Y} \\
   C/Y &= \quad \text{C/Y}
   \end{align*}
   \]

   b) What was the interest earned over 7 years?
10. Consider a $58,331, 30-year loan at 12% interest compounded monthly.

   a) What is the monthly payment for the loan?

   \[
   N = \quad \text{?} \\
   I\% = \quad \text{12} \\
   PV = \quad \text{58,331} \\
   PMT = \quad \text{?} \\
   FV = \quad \text{0} \\
   P/Y = \quad \text{12} \\
   C/Y = \quad \text{12} \\
   \]

   b) How much interest is paid over the 30 years?

   \[
   N = \quad \text{?} \\
   I\% = \quad \text{12} \\
   PV = \quad \text{58,331} \\
   PMT = \quad \text{?} \\
   FV = \quad \text{0} \\
   P/Y = \quad \text{12} \\
   C/Y = \quad \text{12} \\
   \]

   c) How much interest is paid in the first month?

   \[
   N = \quad \text{?} \\
   I\% = \quad \text{12} \\
   PV = \quad \text{58,331} \\
   PMT = \quad \text{?} \\
   FV = \quad \text{0} \\
   P/Y = \quad \text{12} \\
   C/Y = \quad \text{12} \\
   \]
11. If you are 18 years old, deposit $5,000 each year into a traditional IRA for 52 years at 6% interest compounded annually, and retire at age 70, how much money will be in the account upon retirement?

\[ \text{N=} \quad \text{I=} \quad \text{PV=} \quad \text{PMT=} \quad \text{FV=} \quad \text{P/Y=} \quad \text{C/Y=} \]
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\begin{align*}
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I\%=
P\text{V}=
P\text{MT}=
F\text{V}=
P/Y=
C/Y=

b) What was the interest earned over 7 years?
10. Consider a $58,331, 30-year loan at 12% interest compounded monthly.

a) What is the monthly payment for the loan?

\[ \begin{align*}
N &= \_ \\
I\% &= \_ \\
PV &= \_ \\
PMT &= \_ \\
FV &= \_ \\
P/Y &= \_ \\
C/Y &= \_
\end{align*} \]

b) How much interest is paid over the 30 years?

\[ \begin{align*}
N &= \_ \\
I\% &= \_ \\
PV &= \_ \\
PMT &= \_ \\
FV &= \_ \\
P/Y &= \_ \\
C/Y &= \_
\end{align*} \]

c) How much interest is paid in the first month?

\[ \begin{align*}
N &= \_ \\
I\% &= \_ \\
PV &= \_ \\
PMT &= \_ \\
FV &= \_ \\
P/Y &= \_ \\
C/Y &= \_
\end{align*} \]
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