Elements of Modern Mathematics  MAT 183  FALL 2014

FINAL EXAM

VERSION A

NAME (PRINT):

Statement of Ethics regarding this exam
I agree to complete this exam without unauthorized assistance from any person, materials, or device.
Signature: __________________________ Date: 10 December 2014

DIRECTIONS, Read Carefully

• This is a 120 minutes exam.

• This exam consists of 10 questions. Total 110 Points. (Additional 10-Point is extra credit).

• It is your responsibility to make sure that all 9 pages are present.

• The exam is closed book. No extra papers are allowed. If you need extra paper your instructor will give you additional paper. Return the additional paper with your exam paper and write your name.

• Only TI83-TI84 calculators are allowed.

• Turn off your cell phone or any other wireless device or put it into your bag in silent mode.

• Do as Professor Alan does it in class to receive full credit.

• Remember the ethics policy.

• Best Wishes!

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Page 1 of 9
Chapter 5
Let $A, B$ be two sets.

**Inclusion Exclusion principle** $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

**Complement Rule** $n(A) = n(U) - n(A')$.

**De Morgan's Laws**

$$(A \cap B)' = A' \cup B'$$
$$(A \cup B)' = A' \cap B'$$

Chapter 6
Let $A, B$ be two events in a sample space.

**Inclusion Exclusion principle**: $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$.

**Complement Rule**: $Pr(A) = 1 - Pr(A')$.

**Conditional Probability**: If $Pr(B) \neq 0$, then $Pr(A \mid B) = \frac{Pr(A \cap B)}{Pr(B)}$.

**Product Rule**: If $Pr(B) \neq 0$, then $Pr(A \cap B) = Pr(B)Pr(A \mid B)$.

**Independence**: $A$ and $B$ are independent if $Pr(A \cap B) = Pr(A)Pr(B)$.

Chapter 7
**Probability Distribution**
Let $X$ be a random variable with numerical outcomes $x_1, x_2, \ldots, x_N$ with

$Pr(X = x_1) = p_1, Pr(X = x_2) = p_2, \ldots, Pr(X = x_N) = p_N$. Then

$$
\mu = E(X) = x_1p_1 + x_2p_2 + \cdots + x_Np_N.
$$

$$
\text{Variance} = \sigma^2 = (x_1 - \mu)^2p_1 + (x_2 - \mu)^2p_2 + \cdots + (x_N - \mu)^2p_N.
$$

**Binomial Distribution** Let $X$ be a binomial random variable with parameters $n$ and $p$, then

**Binomial Coefficient**: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

**Binomial Probability**: $Pr(X = k) = \binom{n}{k}p^k(1-p)^{n-k}$.

$E(X) = \mu = np, \sigma = \sqrt{np(1-p)}$.

Chapter 10
**Compounded Interest** Present value $P$, future value $F$, interest compounded at a rate $i$ per period

$$
F = (1 + i)^nP\text{ and } P = \frac{F}{(1+i)^n}.
$$

**Simple interest** $r$ is the annual percentage rate and $n$ is the number of years, then

Interest = $nrP$, and $F = P + nrP = (1 + nr)P$. 

Page 2 of 9
1. (10 points) Consider the following matrices.

\[
A = \begin{bmatrix} 1 & 2 & -2 \\ 4 & 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 7 & -7 \\ -5 & 1 & 1 \\ 2 & 2 & 4 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & -3 \\ 1 & 2 & +3 \end{bmatrix}
\]

Perform the indicated matrix calculations (if possible.) If it is not possible explain why it is not possible.

(a) \( A + C = \)

(b) \( A \cdot B = \)

(c) \( B \cdot A = \)

2. (5 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU SOLVE. (Round your answer to 2 decimal places, i.e., nearest cents.)

You are considering the purchase of a condominium to use as a rental property. You estimate that you can rent the condominium for $1490 per month and that taxes, insurance, and maintenance costs will run about $220 per month.

If interest rates are 7% compounded monthly, and you plan a 31-year mortgage. How much money can you borrow and still have the rental income cover all the monthly expenses, i.e., mortgage payment, taxes, insurance, and maintenance coast?

\[
\begin{array}{c}
N = \\
I\% = \\
PV = \\
PMT = \\
FV = \\
P/Y = \\
C/Y = \\
PMT : END
\end{array}
\]
3. (10 points) **YOU MUST SHOW ALL FORMULAS AND CALCULATOR COMMANDS THAT YOU USE!**

Consider the following system of equations.

\[
\begin{align*}
2x + 3y + 5z &= -4 \\
4x - 2y + 7z &= 8 \\
x - y + z &= 3 \\
\end{align*}
\]

(a) Write the corresponding augmented matrix and use Gauss-Jordan Elimination method to transform the matrix to the diagonal form (if possible.) Write the calculator command that you use.

\[
\begin{array}{c|c|c|c}
\hline
\text{Augmented Matrix} & ( & \text{Calculator Command} & \text{Diagonal Form} \\
\hline
\end{array}
\]

(b) Find all solutions.

\[
\begin{align*}
x &= \\
y &= \\
z &= \\
\end{align*}
\]

4. (10 points) **YOU MUST SHOW ALL FORMULAS THAT YOU USE!**

You are to form a 4-digit number from the digits 1, 2, 3, 4, 6, 8, and 9, using each at most once.

(a) Find \( n(S) \).

(b) How many of these numbers are greater than 4000?

(c) What is the probability that a randomly chosen number is greater than 4000?
5. (10 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!** (Round your answer to 4 decimal places.)

(a) 42 percent of all undergraduates at a university are chemistry majors. A random sample of 20 students were chosen. Find the probability that exactly 10 are chemistry majors.

(b) Find the value of $z$ for which the area of the shaded region under the standard normal curve is .28.

(c) Suppose that the lifetimes of a certain light bulb are normally distributed with $\mu = 5000$ hours and $\sigma = 250$. Find the probability that a light bulb will burn out in less than 4800 hours.
6. (15 points) For a certain group of states, it was observed that 70% of the Democratic governors were succeeded by Democrats and 30% by Republicans. Also, 60% of the Republican governors were succeeded by Democrats and 40% by Republicans.

(a) Set up the $2 \times 2$ stochastic matrix with columns and rows labeled D, and R that describes these transitions.

\[
A = \begin{bmatrix}
D & R \\
R & D \\
\end{bmatrix}
\]

(b) Find the stable matrix and the stable distribution. YOU MUST SHOW ALL FORMULAS OR CALCULATOR COMMANDS THAT YOU USE! Give your answers as fractions.

Stable Matrix: \[
\begin{bmatrix}
\quad & \quad \\
\quad & \quad \\
\end{bmatrix}
\] and the stable distribution \[
\begin{bmatrix}
\quad \\
\quad \\
\end{bmatrix}
\]

(c) In the long run, what proportion of the governors will be Democrats?

7. (10 points) Of the registered voters in a certain town, 50% are Democrats, 60% favor a school loan, and 30% are Democrats who favor a school loan. Suppose that a registered voter is selected at random from the town.

(a) What is the conditional probability that the person favors the school loan given that he or she is a Democrat?

(b) What is the conditional probability that the person is a Democrat given that he or she favors the school loan?

(c) What is the probability that the person is not a Democrat and opposes the school loan?
8. (15 points) Consider the following transition diagram.

(a) Write down the transition matrix. Note that this is an absorbing stochastic matrix.

\[
\begin{pmatrix}
\_ & \_ & 0.5 & 0.3 \\
0.1 & \_ & \_ & 0.3 \\
\_ & \_ & \_ & 0.4 \\
\_ & \_ & \_ & \_ \\
\end{pmatrix}
\]

(b) Find R, S and the Fundamental matrix, F. YOU MUST SHOW ALL FORMULAS THAT YOU USE! Give your answers as fractions or round your answer to two decimal places

\[
R = \begin{pmatrix}
\_ & \_ \\
\_ & \_ \\
\_ & \_ \\
\_ & \_ \\
\end{pmatrix}, \quad S = \begin{pmatrix}
\_ & \_ \\
\_ & \_ \\
\_ & \_ \\
\_ & \_ \\
\end{pmatrix}, \quad F = \begin{pmatrix}
\_ & \_ \\
\_ & \_ \\
\_ & \_ \\
\_ & \_ \\
\end{pmatrix}
\]

(c) Find the stable matrix using R, S, F. YOU MUST SHOW ALL FORMULAS THAT YOU USE! Give your answers as fractions or round your answer to two decimal places

\[
\begin{pmatrix}
\_ & \_ & \_ & \_ \\
\_ & \_ & \_ & \_ \\
\_ & \_ & \_ & \_ \\
\_ & \_ & \_ & \_ \\
\end{pmatrix}
\]
9. (10 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU SOLVE.** (Round your answer to 2 decimal places, i.e., nearest cents.)

11,000 dollars is deposited in a savings account at 11% interest compounded quarterly. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) Find the balance after 6 years.

\[
\begin{array}{|l|}
\hline
N = \\
I\% = \\
PV = \\
PMT = \\
FV = \\
P/Y = \\
C/Y = \\
PMT : END \\
\hline
\end{array}
\]

(b) How many quarters are required for the balance to reach $13000? Note that number of quarters are always a whole number.

\[
\begin{array}{|l|}
\hline
N = \\
I\% = \\
PV = \\
PMT = \\
FV = \\
P/Y = \\
C/Y = \\
PMT : END \\
\hline
\end{array}
\]
10. (15 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU SOLVE. (Round your answer to 2 decimal places, i.e., nearest cents.)
Consider a $90270 mortgage to be paid in 23 years, at interest rate 11% compounded monthly, paid at the end of the month. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) How much is the monthly payment?

\[
\begin{align*}
N &= \, \\
I\% &= \, \\
PV &= \\
PMT &= \\
FV &= \\
P/Y &= \\
C/Y &= \\
PMT : END \\
\end{align*}
\]

(b) How much of the first month’s payment is applied to paying off the principal?

\[
\begin{align*}
N &= \, \\
I\% &= \\
PV &= \\
PMT &= \\
FV &= \\
P/Y &= \\
C/Y &= \\
PMT : END \\
\end{align*}
\]

(c) What is the unpaid balance at the end of 11 years?

\[
\begin{align*}
N &= \, \\
I\% &= \\
PV &= \\
PMT &= \\
FV &= \\
P/Y &= \\
C/Y &= \\
PMT : END \\
\end{align*}
\]
Elements of Modern Mathematics  MAT 183  FALL 2014
FINAL EXAM
VERSION B

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**Complement Rule** $n(A) = n(U) - n(A')$.

**De Morgan's Laws**
\[
(A \cap B)' = A' \cup B' \\
(A \cup B)' = A' \cap B'
\]

Chapter 6
Let $A, B$ be two events in a sample space.

**Inclusion Exclusion principle**: $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$.

**Complement Rule** $\Pr(A) = 1 - \Pr(A')$.

**Conditional Probability** If $\Pr(B) \neq 0$, then $\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$.

**Product Rule** If $\Pr(B) \neq 0$, then $\Pr(A \cap B) = \Pr(B) \Pr(A \mid B)$.

**Independence** $A$ and $B$ are independent if $\Pr(A \cap B) = \Pr(A) \Pr(B)$.

Chapter 7
Probability Distribution
Let $X$ be a random variable with numerical outcomes $x_1, x_2, \ldots, x_N$ with

$\Pr(X = x_1) = p_1, \Pr(X = x_2) = p_2, \ldots, \Pr(X = x_N) = p_N$. Then

$\mu = \mathbb{E}(X) = x_1p_1 + x_2p_2 + \cdots + x_Np_N$.

Variance $\sigma^2 = (x_1 - \mu)^2 p_1 + (x_2 - \mu)^2 p_2 + \cdots + (x_N - \mu)^2 p_N$.

**Binomial Distribution** Let $X$ be a binomial random variable with parameters $n$ and $p$, then

**Binomial Coefficient**: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

**Binomial Probability**: $\Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$.

$\mathbb{E}(X) = \mu = np, \sigma = \sqrt{np(1-p)}$.

Chapter 10
Compounded Interest
Present value $P$, future value $F$, interest compounded at a rate $i$ per period

$F = (1+i)^n P$ and $P = \frac{F}{(1+i)^n}$

Simple interest $r$ is the annual percentage rate and $n$ is the number of years, then

Interest $= nrP$, and $F = P + nrP = (1+nr)P$
1. (10 points) Consider the following matrices.

\[
A = \begin{bmatrix}
1 & 2 \\
3 & -1 \\
2 & 4 \\
\end{bmatrix} \quad B = \begin{bmatrix}
1 & -1 & 2 \\
3 & 0 & 2 \\
1 & -1 & 2 \\
\end{bmatrix} \quad C = \begin{bmatrix}
1 & 2 \\
3 & -1 \\
1 & 2 \\
\end{bmatrix}
\]

Perform the indicated matrix calculations (if possible.) If it is not possible explain why it is not possible.

(a) \( A + C = \)

(b) \( A \cdot B = \)

(c) \( B \cdot A = \)

2. (5 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU SOLVE. (Round your answer to 2 decimal places, i.e., nearest cents.)

You are considering the purchase of a condominium to use as a rental property. You estimate that you can rent the condominium for $1580 per month and that taxes, insurance, and maintenance costs will run about $340 per month.

If interest rates are 8% compounded monthly, and you plan a 27-year mortgage. How much money can you borrow and still have the rental income cover all the monthly expenses, i.e., mortgage payment, taxes, insurance, and maintenance coast?

\[
\begin{array}{cccc}
N &=& \\
I\% &=& \\
PV &=& \\
PMT &=& \\
FV &=& \\
P/Y &=& \\
C/Y &=& \\
PMT \ : \ END \\
\end{array}
\]
3. (10 points) YOU MUST SHOW ALL FORMULAS AND CALCULATOR COMMANDS THAT YOU USE!

Consider the following system of equations.

\[
\begin{align*}
5x + y - z &= 9 \\
x - y + 2z &= 7 \\
3x + 2y + z &= 16
\end{align*}
\]

(a) Write the corresponding augmented matrix and use Gauss-Jordan Elimination method to transform the matrix to the diagonal form (if possible.) Write the calculator command that you use.

\[
\begin{bmatrix}
\hline
\hline
\hline
\hline
\hline
\hline
\end{bmatrix}
\]

Augmented Matrix

\[
\begin{bmatrix}
\hline
\hline
\hline
\hline
\hline
\hline
\end{bmatrix}
\]

Calculator Command

\[
\begin{bmatrix}
\hline
\hline
\hline
\hline
\hline
\hline
\end{bmatrix}
\]

Diagonal Form

(b) Find all solutions.

\[
\begin{align*}
x &= \\
y &= \\
z &=
\end{align*}
\]

4. (10 points) YOU MUST SHOW ALL FORMULAS THAT YOU USE!

You are to form a 5-digit number from the digits 1, 2, 3, 4, 6, 8, and 9, using each at most once.

(a) Find \( n(S) \).

(b) How many of these numbers are greater than 20000?

(c) What is the probability that a randomly chosen number is greater than 20000?
5. (10 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!** (Round your answer to 4 decimal places.)

(a) 44 percent of all undergraduates at a university are chemistry majors. A random sample of 21 students were chosen. Find the probability that exactly 11 are chemistry majors.

(b) Find the value of $z$ for which the area of the shaded region under the standard normal curve is .30.

(c) Suppose that the lifetimes of a certain light bulb are normally distributed with $\mu = 5100$ hours and $\sigma = 300$. Find the probability that a light bulb will burn out in more than 5200 hours.
6. (15 points) For a certain group of states, it was observed that 50% of the Democratic governors were succeeded by Democrats and 50% by Republicans. Also, 60% of the Republican governors were succeeded by Democrats and 40% by Republicans.

(a) Set up the $2 \times 2$ stochastic matrix with columns and rows labeled D, and R that describes these transitions.

\[
A = \begin{bmatrix}
D & R \\
D & R
\end{bmatrix}
\]

(b) Find the stable matrix and the stable distribution. **YOU MUST SHOW ALL FORMULAS OR CALCULATOR COMMANDS THAT YOU USE!** Give your answers as fractions.

Stable Matrix: \[
\begin{bmatrix}
\_ & \_ \\
\_ & \_
\end{bmatrix}
\]

and the stable distribution \[
\begin{bmatrix}
\_ \\
\_
\end{bmatrix}
\]

(c) In the long run, what proportion of the governors will be Democrats?

7. (10 points) Of the registered voters in a certain town, 60% are Democrats, 40% favor a school loan, and 20% are Democrats who favor a school loan. Suppose that a registered voter is selected at random from the town.

(a) What is the conditional probability that the person favors the school loan given that he or she is a Democrat?

(b) What is the conditional probability that the person is a Democrat given that he or she favors the school loan?

(c) What is the probability that the person is not a Democrat and opposes the school loan?
8. (15 points) Consider the following transition diagram.

(a) Write down the transition matrix. Note that this is an absorbing stochastic matrix.

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

(b) Find \( R, S \) and the Fundamental matrix, \( F \). YOU MUST SHOW ALL FORMULAS THAT YOU USE! Give your answers as fractions or round your answer to two decimal places.

\[
R = \begin{bmatrix}
1 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}, \quad S = \begin{bmatrix}
1 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}, \quad F = \begin{bmatrix}
1 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}
\]

(c) Find the stable matrix using \( R, S, F \). YOU MUST SHOW ALL FORMULAS THAT YOU USE! Give your answers as fractions or round your answer to two decimal places.

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]
9. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU SOLVE. (Round your answer to 2 decimal places, i.e., nearest cents.)

12,000 dollars is deposited in a savings account at 13% interest compounded quarterly. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) Find the balance after 7 years.

\[
\begin{align*}
N & = \\
I\% & = \\
PV & = \\
PMT & = \\
FV & = \\
P/Y & = \\
C/Y & = \\
PMT : END
\end{align*}
\]

(b) How many quarters are required for the balance to reach $14000? Note that number of quarters are always a whole number.

\[
\begin{align*}
N & = \\
I\% & = \\
PV & = \\
PMT & = \\
FV & = \\
P/Y & = \\
C/Y & = \\
PMT : END
\end{align*}
\]
10. (15 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU SOLVE. (Round your answer to 2 decimal places, i.e., nearest cents.)

Consider a $70,530 mortgage to be paid in 24 years, at interest rate 13% compounded monthly, paid at the end of the month. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) How much is the monthly payment?

\[
\begin{align*}
N & = \\
I\% & = \\
PV & = \\
PMT & = \\
FV & = \\
P/Y & = \\
C/Y & = \\
PMT & \text{ END}
\end{align*}
\]

(b) How much of the first month’s payment is applied to paying off the principal?

\[
\begin{align*}
N & = \\
I\% & = \\
PV & = \\
PMT & = \\
FV & = \\
P/Y & = \\
C/Y & = \\
PMT & \text{ END}
\end{align*}
\]

(c) What is the unpaid balance at the end of 12 years?

\[
\begin{align*}
N & = \\
I\% & = \\
PV & = \\
PMT & = \\
FV & = \\
P/Y & = \\
C/Y & = \\
PMT & \text{ END}
\end{align*}
\]

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Elements of Modern Mathematics  MAT 183  FALL 2014

FINAL EXAM  VERSION C

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Chapter 5
Let $A, B$ be two sets.

Inclusion Exclusion principle $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Complement Rule $n(A) = n(U) - n(A')$.

De Morgan's Laws

\[
(A \cap B)' = A' \cup B' \\
(A \cup B)' = A' \cap B'
\]

Chapter 6
Let $A, B$ be two events in a sample space.

Inclusion Exclusion principle: $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$.

Complement Rule $Pr(A) = 1 - Pr(A')$.

Conditional Probability If $Pr(B) \neq 0$, then $Pr(A | B) = \frac{Pr(A \cap B)}{Pr(B)}$.

Product Rule If $Pr(B) \neq 0$, then $Pr(A \cap B) = Pr(B)Pr(A | B)$.

Independence $A$ and $B$ are independent if $Pr(A \cap B) = Pr(A)Pr(B)$.

Chapter 7
Probability Distribution
Let $X$ be a random variable with numerical outcomes $x_1, x_2, \ldots, x_N$ with

$Pr(X = x_1) = p_1, Pr(X = x_2) = p_2, \ldots, Pr(X = x_N) = p_N$. Then

$\mu = E(X) = x_1p_1 + x_2p_2 + \cdots + x_Np_N$.

Variance $\sigma^2 = (x_1 - \mu)^2p_1 + (x_2 - \mu)^2p_2 + \cdots + (x_N - \mu)^2p_N$.

Binomial Distribution Let $X$ be a binomial random variable with parameters $n$ and $p$, then

Binomial Coefficient: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Binomial Probability: $Pr(X = k) = \binom{n}{k}p^k(1-p)^{n-k}$.

$E(X) = \mu = np$, $\sigma = \sqrt{np(1-p)}$.

Chapter 10
Compounded Interest Present value $P$, future value $F$, interest compounded at a rate $i$ per period

$F = (1+i)^nP$ and $P = \frac{F}{(1+i)^n}$

Simple interest $r$ is the annual percentage rate and $n$ is the number of years, then

Interest $= nrP$, and $F = P + nrP = (1+nr)P$
1. (10 points) Consider the following matrices.

\[
A = \begin{bmatrix}
-1 & 1 & -2 \\
-3 & 2 & 0
\end{bmatrix} \quad B = \begin{bmatrix}
-1 & 1 & -2 \\
-3 & 2 & 0 \\
1 & -1 & 2
\end{bmatrix} \quad C = \begin{bmatrix}
-3 & 2 & 0 \\
-1 & 1 & -2
\end{bmatrix}
\]

Perform the indicated matrix calculations (if possible.) If it is not possible explain why it is not possible.

(a) \(A + C = \)

(b) \(A \times B = \)

(c) \(B \times A = \)

2. (5 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU SOLVE. (Round your answer to 2 decimal places, i.e., nearest cents.)

You are considering the purchase of a condominium to use as a rental property. You estimate that you can rent the condominium for $1670 per month and that taxes, insurance, and maintenance costs will run about $230 per month.

If interest rates are 10% compounded monthly, and you plan a 28-year mortgage. How much money can you borrow and still have the rental income cover all the monthly expenses, i.e., mortgage payment, taxes, insurance, and maintenance coast?

\[
\begin{array}{c}
N = \\
I\% = \\
PV = \\
PMT = \\
FV = \\
P/Y = \\
C/Y = \\
PMT : END
\end{array}
\]
3. (10 points) YOU MUST SHOW ALL FORMULAS AND CALCULATOR COMMANDS THAT YOU USE!

Consider the following system of equations.

\[
\begin{align*}
    x + 4y - 2z &= 6 \\
    -2x - 3y &= -8 \\
    x + 4y + 5z &= -1
\end{align*}
\]

(a) Write the corresponding augmented matrix and use Gauss-Jordan Elimination method to transform the matrix to the diagonal form (if possible.) Write the calculator command that you use.

\[
\begin{bmatrix}
    -1 & 4 & -2 & | & 6 \\
    -2 & -3 & 0 & | & -8 \\
    1 & 4 & 5 & | & -1
\end{bmatrix}
\]

(b) Find all solutions.

\[
\begin{align*}
    x &= \\
    y &= \\
    z &= 
\end{align*}
\]

4. (10 points) YOU MUST SHOW ALL FORMULAS THAT YOU USE!

You are to form a 4-digit number from the digits 1, 2, 3, 4, 6, 8, and 9, using each at most once.

(a) Find \(n(S)\).

(b) How many of these numbers are greater than 6000?

(c) What is the probability that a randomly chosen number is greater than 6000?
5. (10 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!** (Round your answer to 4 decimal places.)

(a) 45 percent of all undergraduates at a university are chemistry majors. A random sample of 22 students were chosen. Find the probability that exactly 12 are chemistry majors.

(b) Find the value of $z$ for which the area of the shaded region under the standard normal curve is .32. 

(c) Suppose that the lifetimes of a certain light bulb are normally distributed with $\mu = 5200$ hours and $\sigma = 350$. Find the probability that a light bulb will burn out in less than 5050 hours.
6. (15 points) For a certain group of states, it was observed that 60% of the Democratic governors were succeeded by Democrats and 40% by Republicans. Also, 50% of the Republican governors were succeeded by Democrats and 50% by Republicans.

(a) Set up the 2 x 2 stochastic matrix with columns and rows labeled D, and R that describes these transitions.

\[ A = \begin{bmatrix} D & R \\ D & R \end{bmatrix} \]

(b) Find the stable matrix and the stable distribution. YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! Give your answers as fractions

Stable Matrix: \[ \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix} \] and the stable distribution \[ \begin{bmatrix} \_ \end{bmatrix} \]

(c) In the long run, what proportion of the governors will be Democrats?

7. (10 points) Of the registered voters in a certain town, 70% are Democrats, 60% favor a school loan, and 40% are Democrats who favor a school loan. Suppose that a registered voter is selected at random from the town.

(a) What is the conditional probability that the person favors the school loan given that he or she is a Democrat?

(b) What is the conditional probability that the person is a Democrat given that he or she favors the school loan?

(c) What is the probability that the person is not a Democrat and opposes the school loan?
8. (15 points) Consider the following transition diagram.

(a) Write down the transition matrix. Note that this is an absorbing stochastic matrix.

\[
\begin{bmatrix}
\_ & \_ & \_ & \_ \\
\_ & \_ & \_ & \_ \\
\_ & \_ & \_ & \_ \\
\_ & \_ & \_ & \_ \\
\end{bmatrix}
\]

(b) Find R, S and the Fundamental matrix, F. YOU MUST SHOW ALL FORMULAS THAT YOU USE! Give your answers as fractions or round your answer to two decimal places.

\[ R = \begin{bmatrix} \_ & \_ \end{bmatrix}, \quad S = \begin{bmatrix} \_ & \_ \end{bmatrix}, \quad F = \begin{bmatrix} \_ & \_ \end{bmatrix} \]

(c) Find the stable matrix using R, S, F. YOU MUST SHOW ALL FORMULAS THAT YOU USE! Give your answers as fractions or round your answer to two decimal places.

\[
\begin{bmatrix}
\_ & \_ & \_ & \_ \\
\_ & \_ & \_ & \_ \\
\_ & \_ & \_ & \_ \\
\_ & \_ & \_ & \_ \\
\end{bmatrix}
\]
9. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU SOLVE. (Round your answer to 2 decimal places, i.e., nearest cents.)

13,000 dollars is deposited in a savings account at 14% interest compounded quarterly. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) Find the balance after 8 years.

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(b) How many quarters are required for the balance to reach $15,000? Note that number of quarters are always a whole number.

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10. (15 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU SOLVE.** (Round your answer to 2 decimal places, i.e., nearest cents.)

Consider a $50,228 mortgage to be paid in 27 years, at interest rate 14% compounded monthly, paid at the end of the month. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) How much is the monthly payment?

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(b) How much of the first month’s payment is applied to paying off the principal?

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(c) What is the unpaid balance at the end of 13 years?

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Elements of Modern Mathematics  MAT 183  FALL 2014
FINAL EXAM
VERSION D

NAME (PRINT):

Statement of Ethics regarding this exam
I agree to complete this exam without unauthorized assistance from any person, materials, or device.
Signature: ___________________________ Date: 10 December 2014

DIRECTIONS, Read Carefully

• This is a 120 minutes exam.
• This exam consists of 10 questions. Total 110 Points. (Additional 10-Point is extra credit).
• It is your responsibility to make sure that all 9 pages are present.
• The exam is closed book. No extra papers are allowed. If you need extra paper your instructor will give you additional paper. Return the additional paper with your exam paper and write your name.
• Only TI83-TI84 calculators are allowed.
• Turn off your cell phone or any other wireless device or put it into your bag in silent mode.
• Do as Professor Alan does it in class to receive full credit.
• Remember the ethics policy.
• Best Wishes!

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Chapter 5
Let $A, B$ be two sets.

Inclusion Exclusion principle  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Complement Rule  $n(A) = n(U) - n(A')$.

De Morgan's Laws

\[
(A \cap B)' = A' \cup B' \\
(A \cup B)' = A' \cap B'
\]

Chapter 6
Let $A, B$ be two events in a sample space.

Inclusion Exclusion principle: $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$.

Complement Rule  $\Pr(A) = 1 - \Pr(A')$.

Conditional Probability  If $\Pr(B) \neq 0$, then $\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$.

Product Rule  If $\Pr(B) \neq 0$, then $\Pr(A \cap B) = \Pr(B) \Pr(A \mid B)$.

Independence  $A$ and $B$ are independent if $\Pr(A \cap B) = \Pr(A) \Pr(B)$.

Chapter 7
Probability Distribution
Let $X$ be a random variable with numerical outcomes $x_1, x_2, \ldots, x_N$ with

\[
\Pr(X = x_1) = p_1, \Pr(X = x_2) = p_2, \ldots, \Pr(X = x_N) = p_N.
\]

Then

\[
\mu = E(X) = x_1 p_1 + x_2 p_2 + \cdots + x_N p_N.
\]

Variance $\sigma^2 = (x_1 - \mu)^2 p_1 + (x_2 - \mu)^2 p_2 + \cdots + (x_N - \mu)^2 p_N$.

Binomial Distribution
Let $X$ be a binomial random variable with parameters $n$ and $p$, then

Binomial Coefficient: \( \binom{n}{k} = \frac{n!}{k!(n-k)!} \).

Binomial Probability: \( \Pr(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} \).

\( E(X) = \mu = np, \sigma = \sqrt{np(1-p)} \).

Chapter 10
Compounded Interest
Present value $P$, future value $F$, interest compounded at a rate $i$ per period

\[
F = (1+i)^n P \quad \text{and} \quad P = \frac{F}{(1+i)^n}
\]

Simple interest $r$ is the annual percentage rate and $n$ is the number of years, then

Interest $= nrP$, and $F = P + nrP = (1+nr)P$.
1. (10 points) Consider the following matrices.

\[ A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \\ 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 2 \\ -3 & 0 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 2 & 4 \end{bmatrix} \]

Perform the indicated matrix calculations (if possible.) If it is not possible explain why it is not possible.

(a) \( A + C = \)

(b) \( A \cdot B = \)

(c) \( B \cdot A = \)

2. (5 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU SOLVE. (Round your answer to 2 decimal places, i.e., nearest cents.)

You are considering the purchase of a condominium to use as a rental property. You estimate that you can rent the condominium for $1760 per month and that taxes, insurance, and maintenance costs will run about $240 per month.

If interest rates are 11% compounded monthly, and you plan a 29-year mortgage. How much money can you borrow and still have the rental income cover all the monthly expenses, i.e., mortgage payment, taxes, insurance, and maintenance cost?

\[
\begin{array}{c}
N = \\
I\% = \\
PV = \\
PMT = \\
PV = \\
P/Y = \\
C/Y = \\
PMT : \text{ END} \\
\end{array}
\]
3. (10 points) **YOU MUST SHOW ALL FORMULAS AND CALCULATOR COMMANDS THAT YOU USE!**

Consider the following system of equations.

\[
\begin{align*}
2x - 3y &= 9 \\
-x + 4y + 4z &= -8 \\
-2x + 3y - 2z &= -11
\end{align*}
\]

(a) Write the corresponding augmented matrix and use Gauss-Jordan Elimination method to transform the matrix to the diagonal form (if possible.) **Write the calculator command that you use.**

```
Augmented Matrix

(- - - - - )

(- - - - - )

Calculator Command

Diagonal Form

(- - - - - )

(- - - - - )
```

(b) Find all solutions.

\[x = \] \\
\[y = \] \\
\[z = \]

4. (10 points) **YOU MUST SHOW ALL FORMULAS THAT YOU USE!**

You are to form a 5-digit number from the digits 1, 2, 3, 4, 5, 6, 8, and 9, using each at most once.

(a) Find \( n(S) \).

(b) How many of these numbers are greater than 30000?

(c) What is the probability that a randomly chosen number is greater than 30000?
5. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! (Round your answer to 4 decimal places.)

(a) 46 percent of all undergraduates at a university are chemistry majors. A random sample of 23 students were chosen. Find the probability that exactly 14 are chemistry majors.

(b) Find the value of $z$ for which the area of the shaded region under the standard normal curve is .34.

(c) Suppose that the lifetimes of a certain light bulb are normally distributed with $\mu = 5250$ hours and $\sigma = 400$. Find the probability that a light bulb will burn out in less than 5100 hours.
6. (15 points) For a certain group of states, it was observed that 70% of the Democratic governors were succeeded by Democrats and 30% by Republicans. Also, 20% of the Republican governors were succeeded by Democrats and 80% by Republicans.

(a) Set up the $2 \times 2$ stochastic matrix with columns and rows labeled D, and R that describes these transitions.

$$A = \begin{bmatrix} D & R \\ \_ & \_ \end{bmatrix}$$

(b) Find the stable matrix and the stable distribution. YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! Give your answers as fractions

Stable Matrix: $\begin{bmatrix} \_ & \_ \end{bmatrix}$ and the stable distribution $\begin{bmatrix} \_ \end{bmatrix}$

(c) In the long run, what proportion of the governors will be Democrats?

7. (10 points) Of the registered voters in a certain town, 60% are Democrats, 60% favor a school loan, and 40% are Democrats who favor a school loan. Suppose that a registered voter is selected at random from the town.

(a) What is the conditional probability that the person favors the school loan given that he or she is a Democrat?

(b) What is the conditional probability that the person is a Democrat given that he or she favors the school loan?

(c) What is the probability that the person is not a Democrat and opposes the school loan?
8. (15 points) Consider the following transition diagram.

(a) Write down the transition matrix. Note that this is an absorbing stochastic matrix.

\[
\begin{pmatrix}
0 & 0.1 & 0.3 & 0.1 \\
0.5 & 0.3 & 0.3 & 0.4 \\
0.5 & 0.3 & 0.3 & 0.4 \\
0.5 & 0.3 & 0.3 & 0.4 \\
\end{pmatrix}
\]

(b) Find R, S and the Fundamental matrix, F. YOU MUST SHOW ALL FORMULAS THAT YOU USE! Give your answers as fractions or round your answer to two decimal places.

\[
R = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}, \\
S = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}, \\
F = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

(c) Find the stable matrix using R, S, F. YOU MUST SHOW ALL FORMULAS THAT YOU USE! Give your answers as fractions or round your answer to two decimal places.

\[
\begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\]
9. (10 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU SOLVE.** (Round your answer to 2 decimal places, i.e., nearest cents.)

14,000 dollars is deposited in a savings account at 16% interest compounded quarterly. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) Find the balance after 9 years.

\[
\begin{array}{l}
N = \\
I\% = \\
PV = \\
PMT = \\
FV = \\
P/Y = \\
C/Y = \\
PMT : \text{END}
\end{array}
\]

(b) How many quarters are required for the balance to reach $16,000? Note that number of quarters are always a whole number.

\[
\begin{array}{l}
N = \\
I\% = \\
PV = \\
PMT = \\
FV = \\
P/Y = \\
C/Y = \\
PMT : \text{END}
\end{array}
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Consider a $55,593 mortgage to be paid in 28 years, at interest rate 16% compounded monthly, paid at the end of the month. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) How much is the monthly payment?

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(b) How much of the first month's payment is applied to paying off the principal?

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(c) What is the unpaid balance at the end of 14 years?

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Elements of Modern Mathematics       MAT 183       FALL 2014

FINAL EXAM

VERSION E

NAME (PRINT):

Statement of Ethics regarding this exam
I agree to complete this exam without unauthorized assistance from any person, materials, or device.
Signature: __________________ Date: 10 December 2014

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Chapter 5
Let $A, B$ be two sets.

Inclusion Exclusion principle  
\[ n(A \cup B) = n(A) + n(B) - n(A \cap B). \]

Complement Rule  
\[ n(A) = n(U) - n(A'). \]

De Morgan's Laws
\[
\begin{align*}
(A \cap B)' &= A' \cup B' \\
(A \cup B)' &= A' \cap B'
\end{align*}
\]

Chapter 6
Let $A, B$ be two events in a sample space.

Inclusion Exclusion principle:  
\[ \Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B). \]

Complement Rule  
\[ \Pr(A) = 1 - \Pr(A'). \]

Conditional Probability  
If $\Pr(B) \neq 0$, then  
\[ \Pr(A | B) = \frac{\Pr(A \cap B)}{\Pr(B)}. \]

Product Rule  
If $\Pr(B) \neq 0$, then  
\[ \Pr(A \cap B) = \Pr(B) \Pr(A | B). \]

Independence  
$A$ and $B$ are independent if  
\[ \Pr(A \cap B) = \Pr(A) \Pr(B). \]

Chapter 7
Probability Distribution
Let $X$ be a random variable with numerical outcomes $x_1, x_2, \ldots, x_N$ with  
\[ \Pr(X = x_1) = p_1, \Pr(X = x_2) = p_2, \ldots, \Pr(X = x_N) = p_N. \]  
Then  
\[
\begin{align*}
\mu &= E(X) = x_1 p_1 + x_2 p_2 + \cdots + x_N p_N. \\
\text{Variance} &= \sigma^2 = (x_1 - \mu)^2 p_1 + (x_2 - \mu)^2 p_2 + \cdots + (x_N - \mu)^2 p_N.
\end{align*}
\]

Binomial Distribution  
Let $X$ be a binomial random variable with parameters $n$ and $p$, then  
\[
\begin{align*}
\text{Binomial Coefficient:} & \quad \binom{n}{k} = \frac{n!}{k!(n-k)!}. \\
\text{Binomial Probability:} & \quad \Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}. \\
E(X) &= \mu = np, \sigma = \sqrt{np(1-p)}. \]

Chapter 10
Compounded Interest  
Present value $P$, future value $F$, interest compounded at a rate $i$ per period  
\[ F = (1+i)^n P \text{ and } P = \frac{F}{(1+i)^n}. \]

Simple interest $r$ is the annual percentage rate and $n$ is the number of years, then  
\[ \text{Interest} = nrP, \text{ and } F = P + nrP = (1+nr)P. \]
1. (10 points) Consider the following matrices.

\[
A = \begin{bmatrix} 1 & 2 & -2 \\ 4 & 1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 7 & -7 \\ -5 & 1 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & -3 \\ 1 & 2 & +3 \end{bmatrix}
\]

Perform the indicated matrix calculations (if possible.) If it is not possible explain why it is not possible.

(a) \( A + C = \)

(b) \( A \cdot B = \)

(c) \( B \cdot A = \)

2. (5 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU SOLVE. (Round your answer to 2 decimal places, i.e., nearest cents.)

You are considering the purchase of a condominium to use as a rental property. You estimate that you can rent the condominium for $1490 per month and that taxes, insurance, and maintenance costs will run about $220 per month.

If interest rates are 7% compounded monthly, and you plan a 31-year mortgage. How much money can you borrow and still have the rental income cover all the monthly expenses, i.e., mortgage payment, taxes, insurance, and maintenance coast?

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<tr>
<th>( N = )</th>
<th>( I% = )</th>
<th>( PV = )</th>
<th>( PMT = )</th>
<th>( FV = )</th>
<th>( P/Y = )</th>
<th>( C/Y = )</th>
<th>( PMT : ) END</th>
</tr>
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</table>

Page 3 of 9
3. (10 points) **YOU MUST SHOW ALL FORMULAS AND CALCULATOR COMMANDS THAT YOU USE!**

Consider the following system of equations.

\[
\begin{align*}
2x + 3y + 5z &= -4 \\
4x - 2y + 7z &= 8 \\
x - y + z &= 3
\end{align*}
\]

(a) Write the corresponding augmented matrix and use Gauss-Jordan Elimination method to transform the matrix to the diagonal form (if possible.) **Write the calculator command that you use.**

**Augmented Matrix**

\[
\begin{bmatrix}
- & - & - & - \\
- & - & - & - \\
- & - & - & - \\
\end{bmatrix}
\]

**Calculator Command**

\[
\begin{bmatrix}
- & - & - & - \\
- & - & - & - \\
- & - & - & - \\
\end{bmatrix}
\]

**Diagonal Form**

(b) Find all solutions.

\[
\begin{align*}
x &= \\
y &= \\
z &= 
\end{align*}
\]

4. (10 points) **YOU MUST SHOW ALL FORMULAS THAT YOU USE!**

You are to form a 4-digit number from the digits 1, 2, 3, 4, 6, 8, and 9, using each at most once.

(a) Find \( n(S) \).

(b) How many of these numbers are greater than 4000?

(c) What is the probability that a randomly chosen number is greater than 4000?
5. (10 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!** (Round your answer to 4 decimal places.)

(a) 42 percent of all undergraduates at a university are chemistry majors. A random sample of 20 students were chosen. Find the probability that exactly 10 are chemistry majors.

(b) Find the value of $z$ for which the area of the shaded region under the standard normal curve is .28.

(c) Suppose that the lifetimes of a certain light bulb are normally distributed with $\mu = 5000$ hours and $\sigma = 250$. Find the probability that a light bulb will burn out in less than 4800 hours.
6. (15 points) For a certain group of states, it was observed that 70% of the Democratic governors were succeeded by Democrats and 30% by Republicans. Also, 60% of the Republican governors were succeeded by Democrats and 40% by Republicans.

(a) Set up the $2 \times 2$ stochastic matrix with columns and rows labeled D, and R that describes these transitions.

$$A = \begin{bmatrix} D & R \\ D & R \end{bmatrix}$$

(b) Find the stable matrix and the stable distribution. YOU MUST SHOW ALL FORMULAS OR CALCULATOR COMMANDS THAT YOU USE! Give your answers as fractions

Stable Matrix: $\begin{bmatrix} \quad \quad \\ \quad \quad \end{bmatrix}$ and the stable distribution $\begin{bmatrix} \quad \quad \end{bmatrix}$

(c) In the long run, what proportion of the governors will be Democrats?

7. (10 points) Of the registered voters in a certain town, 50% are Democrats, 60% favor a school loan, and 30% are Democrats who favor a school loan. Suppose that a registered voter is selected at random from the town.

(a) What is the conditional probability that the person favors the school loan given that he or she is a Democrat?

(b) What is the conditional probability that the person is a Democrat given that he or she favors the school loan?

(c) What is the probability that the person is not a Democrat and opposes the school loan?
8. (15 points) Consider the following transition diagram.

(a) Write down the transition matrix. Note that this is an absorbing stochastic matrix.

\[
\begin{bmatrix}
 & & & \\
 & & & \\
 & & & \\
\end{bmatrix}
\]

(b) Find \( R \), \( S \) and the Fundamental matrix, \( F \). YOU MUST SHOW ALL FORMULAS THAT YOU USE! Give your answers as fractions or round your answer to two decimal places

\[
R = \begin{bmatrix}
 & \\
 & \\
\end{bmatrix}, \quad S = \begin{bmatrix}
 & \\
 & \\
\end{bmatrix}, \quad F = \begin{bmatrix}
 & \\
 & \\
\end{bmatrix}
\]

(c) Find the stable matrix using \( R, S, F \). YOU MUST SHOW ALL FORMULAS THAT YOU USE! Give your answers as fractions or round your answer to two decimal places

\[
\begin{bmatrix}
 & & & \\
 & & & \\
 & & & \\
\end{bmatrix}
\]
9. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU SOLVE. (Round your answer to 2 decimal places, i.e., nearest cents.)

11,000 dollars is deposited in a savings account at 11% interest compounded quarterly. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) Find the balance after 6 years.

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<tr>
<th>N</th>
<th>I%</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
<th>P/Y</th>
<th>C/Y</th>
<th>PMT : END</th>
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(b) How many quarters are required for the balance to reach $13000? Note that number of quarters are always a whole number.

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<th>I%</th>
<th>PV</th>
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Consider a $90270 mortgage to be paid in 23 years, at interest rate 11% compounded monthly, paid at the end of the month. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) How much is the monthly payment?

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(b) How much of the first month’s payment is applied to paying off the principal?

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(c) What is the unpaid balance at the end of 11 years?

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<th>I%</th>
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<th>FV</th>
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Page 9 of 9
Statement of Ethics regarding this exam
I agree to complete this exam without unauthorized assistance from any person, materials, or device.
Signature: __________________________ Date: 10 December 2014

DIRECTIONS, Read Carefully

- This is a 120 minutes exam.

- This exam consists of 10 questions. Total 110 Points. (Additional 10-Point is extra credit).

- It is your responsibility to make sure that all 9 pages are present.

- The exam is closed book. No extra papers are allowed. If you need extra paper your instructor will give you additional paper. Return the additional paper with your exam paper and write your name.

- Only TI83-TI84 calculators are allowed.

- Turn off your cell phone or any other wireless device or put it into your bag in silent mode.

- Do as Professor Alan does it in class to receive full credit.

- Remember the ethics policy.

- Best Wishes!

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Chapter 5
Let $A, B$ be two sets.

**Inclusion Exclusion principle** $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

**Complement Rule** $n(A) = n(U) - n(A')$.

**De Morgan’s Laws**

\[
(A \cap B)' = A' \cup B' \\
(A \cup B)' = A' \cap B'
\]

Chapter 6
Let $A, B$ be two events in a sample space.

**Inclusion Exclusion principle** $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$.

**Complement Rule** $\Pr(A) = 1 - \Pr(A')$.

**Conditional Probability** If $\Pr(B) \neq 0$, then $\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$.

**Product Rule** If $\Pr(B) \neq 0$, then $\Pr(A \cap B) = \Pr(B) \Pr(A \mid B)$.

**Independence** $A$ and $B$ are independent if $\Pr(A \cap B) = \Pr(A) \Pr(B)$.

Chapter 7
**Probability Distribution**
Let $X$ be a random variable with numerical outcomes $x_1, x_2, \ldots, x_N$ with
$\Pr(X = x_1) = p_1, \Pr(X = x_2) = p_2, \ldots, \Pr(X = x_N) = p_N$. Then

\[
\mu = E(X) = x_1 p_1 + x_2 p_2 + \cdots + x_N p_N.
\]

Variance $\sigma^2 = (x_1 - \mu)^2 p_1 + (x_2 - \mu)^2 p_2 + \cdots + (x_N - \mu)^2 p_N$.

**Binomial Distribution** Let $X$ be a binomial random variable with parameters $n$ and $p$, then

Binomial Coefficient: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Binomial Probability: $\Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$.

$E(X) = \mu = np$, $\sigma = \sqrt{np(1-p)}$.

Chapter 10
**Compounded Interest** Present value $P$, future value $F$, interest compounded at a rate $i$ per period

$F = (1+i)^n P$ and $P = \frac{F}{(1+i)^n}$

**Simple interest** $r$ is the annual percentage rate and $n$ is the number of years, then

Interest = $nrP$, and $F = P + nrP = (1 + nr)P$
1. (10 points) Consider the following matrices.

\[
A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}
\]

Perform the indicated matrix calculations (if possible.) If it is not possible explain why it is not possible.

(a) \( A + C = \)

(b) \( A \cdot B = \)

(c) \( B \cdot A = \)

2. (5 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU SOLVE. (Round your answer to 2 decimal places, i.e., nearest cents.)

You are considering the purchase of a condominium to use as a rental property. You estimate that you can rent the condominium for $1580 per month and that taxes, insurance, and maintenance costs will run about $340 per month.

If interest rates are 8% compounded monthly, and you plan a 27-year mortgage. How much money can you borrow and still have the rental income cover all the monthly expenses, i.e., mortgage payment, taxes, insurance, and maintenance costs?

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<th>( I% )</th>
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<th>( FV )</th>
<th>( P/Y )</th>
<th>( C/Y )</th>
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3. (10 points) YOU MUST SHOW ALL FORMULAS AND CALCULATOR COMMANDS THAT YOU USE!
Consider the following system of equations.

\[
\begin{align*}
5x + y - z &= 9 \\
x - y + 2z &= 7 \\
3x + 2y + z &= 16
\end{align*}
\]

(a) Write the corresponding augmented matrix and use Gauss-Jordan Elimination method to transform the matrix to the diagonal form (if possible.) Write the calculator command that you use.

\[
\begin{pmatrix}
\text{Augmented Matrix} \\
\text{Calculator Command} \\
\text{Diagonal Form}
\end{pmatrix}
\]

(b) Find all solutions.

\[
\begin{align*}
x &= \\
y &= \\
z &= 
\end{align*}
\]

4. (10 points) YOU MUST SHOW ALL FORMULAS THAT YOU USE!
You are to form a 5-digit number from the digits 1, 2, 3, 4, 6, 8, and 9, using each at most once.

(a) Find \(n(S)\).

(b) How many of these numbers are greater than 20000?

(c) What is the probability that a randomly chosen number is greater than 20000?
5. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! (Round your answer to 4 decimal places.)

(a) 44 percent of all undergraduates at a university are chemistry majors. A random sample of 21 students were chosen. Find the probability that exactly 11 are chemistry majors.

(b) Find the value of \( z \) for which the area of the shaded region under the standard normal curve is .30.

(c) Suppose that the lifetimes of a certain light bulb are normally distributed with \( \mu = 5100 \) hours and \( \sigma = 300 \). Find the probability that a light bulb will burn out in more than 5200 hours.
6. (15 points) For a certain group of states, it was observed that 50% of the Democratic governors were succeeded by Democrats and 50% by Republicans. Also, 60% of the Republican governors were succeeded by Democrats and 40% by Republicans.

(a) Set up the $2 \times 2$ stochastic matrix with columns and rows labeled D, and R that describes these transitions.

$$A = \begin{bmatrix} D & R \\ R & D \end{bmatrix}$$

(b) Find the stable matrix and the stable distribution. YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! Give your answers as fractions

Stable Matrix: $\begin{bmatrix} \_ \_ \\ \_ \_ \end{bmatrix}$ and the stable distribution $\begin{bmatrix} \_ \_ \end{bmatrix}$

(c) In the long run, what proportion of the governors will be Democrats?

7. (10 points) Of the registered voters in a certain town, 60% are Democrats, 40% favor a school loan, and 20% are Democrats who favor a school loan. Suppose that a registered voter is selected at random from the town.

(a) What is the conditional probability that the person favors the school loan given that he or she is a Democrat?

(b) What is the conditional probability that the person is a Democrat given that he or she favors the school loan?

(c) What is the probability that the person is not a Democrat and opposes the school loan?
8. (15 points) Consider the following transition diagram.

(a) Write down the transition matrix. Note that this is an absorbing stochastic matrix.

\[
\begin{bmatrix}
0.2 & 0.3 \\
0.5 & 0.1 \\
0.3 & 0.4 \\
0 & 1
\end{bmatrix}
\]

(b) Find R, S and the Fundamental matrix, F. YOU MUST SHOW ALL FORMULAS THAT YOU USE! Give your answers as fractions or round your answer to two decimal places.

\[
R = \begin{bmatrix}
0.2 & 0.3 \\
0.5 & 0.1 \\
0.3 & 0.4 \\
0 & 1
\end{bmatrix},
S = \begin{bmatrix}
0.2 & 0.3 \\
0.5 & 0.1 \\
0.3 & 0.4 \\
0 & 1
\end{bmatrix},
F = \begin{bmatrix}
0.2 & 0.3 \\
0.5 & 0.1 \\
0.3 & 0.4 \\
0 & 1
\end{bmatrix}
\]

(c) Find the stable matrix using R, S, F. YOU MUST SHOW ALL FORMULAS THAT YOU USE! Give your answers as fractions or round your answer to two decimal places.

\[
\begin{bmatrix}
0.2 & 0.3 \\
0.5 & 0.1 \\
0.3 & 0.4 \\
0 & 1
\end{bmatrix}
\]
9. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT
YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED
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SOLVE. (Round your answer to 2 decimal places, i.e., nearest cents.)
12,000 dollars is deposited in a savings account at 13% interest compounded quarterly. (Round
your answer to 2 decimal places, i.e., nearest cents.)

(a) Find the balance after 7 years.

- N =
- I% =
- PV =
- PMT =
- FV =
- P/Y =
- C/Y =
- PMT : END

(b) How many quarters are required for the balance to reach $14000? Note that number of
quarters are always a whole number.

- N =
- I% =
- PV =
- PMT =
- FV =
- P/Y =
- C/Y =
- PMT : END
10. (15 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT
YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED
IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU
SOLVE. (Round your answer to 2 decimal places, i.e., nearest cents.)
Consider a $70,530 mortgage to be paid in 24 years, at interest rate 13% compounded monthly,
paid at the end of the month. (Round your answer to 2 decimal places, i.e., nearest
cents.)

(a) How much is the monthly payment?

N = 
I% = 
PV = 
PMT = 
FV = 
P/Y = 
C/Y = 
PMT : END

(b) How much of the first month's payment is applied to paying off the principal?

N = 
I% = 
PV = 
PMT = 
FV = 
P/Y = 
C/Y = 
PMT : END

(c) What is the unpaid balance at the end of 12 years?

N = 
I% = 
PV = 
PMT = 
FV = 
P/Y = 
C/Y = 
PMT : END
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I agree to complete this exam without unauthorized assistance from any person, materials, or device.
Signature: ___________________________ Date: 10 December 2014

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Let $A, B$ be two sets.

**Inclusion Exclusion principle** $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

**Complement Rule** $n(A) = n(U) - n(A')$.

**De Morgan's Laws**
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(A \cap B)' = A' \cup B' \\
(A \cup B)' = A' \cap B'
\]

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Let $A, B$ be two events in a sample space.

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**Independence** $A$ and $B$ are independent if $\Pr(A \cap B) = \Pr(A) \Pr(B)$.

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Let $X$ be a random variable with numerical outcomes $x_1, x_2, \ldots, x_N$ with
\[
\Pr(X = x_1) = p_1, \Pr(X = x_2) = p_2, \ldots, \Pr(X = x_N) = p_N.
\]
Then
\[
\mu = E(X) = x_1p_1 + x_2p_2 + \cdots + x_Np_N.
\]

**Variance** $\sigma^2 = (x_1 - \mu)^2p_1 + (x_2 - \mu)^2p_2 + \cdots + (x_N - \mu)^2p_N$.

**Binomial Distribution** Let $X$ be a binomial random variable with parameters $n$ and $p$, then
\[
\binom{n}{k} = \frac{n!}{k!(n-k)!}.
\]

**Binomial Probability** $\Pr(X = k) = \binom{n}{k}p^k(1-p)^{n-k}$.

$E(X) = \mu = np$, $\sigma = \sqrt{np(1-p)}$.

Chapter 10
**Compounded Interest** Present value $P$, future value $F$, interest compounded at a rate $i$ per period
\[
F = (1 + i)^n P \quad \text{and} \quad P = \frac{F}{(1 + i)^n}
\]

**Simple interest** $r$ is the annual percentage rate and $n$ is the number of years, then
\[
\text{Interest} = nrP, \quad \text{and} \quad F = P + nrP = (1 + nr)P
\]
1. (10 points) Consider the following matrices.

\[ A = \begin{bmatrix} -1 & 1 & -2 \\ -3 & 2 & 0 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 1 & -2 \\ -3 & 2 & 0 \\ 1 & -1 & 2 \end{bmatrix} \quad C = \begin{bmatrix} -3 & 2 & 0 \\ -1 & 1 & -2 \end{bmatrix} \]

Perform the indicated matrix calculations (if possible.) If it is not possible explain why it is not possible.

(a) \( A + C = \)

(b) \( A \cdot B = \)

(c) \( B \cdot A = \)

2. (5 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU SOLVE. (Round your answer to 2 decimal places, i.e., nearest cents.)

You are considering the purchase of a condominium to use as a rental property. You estimate that you can rent the condominium for $1670 per month and that taxes, insurance, and maintenance costs will run about $230 per month.

If interest rates are 10% compounded monthly, and you plan a 28-year mortgage. How much money can you borrow and still have the rental income cover all the monthly expenses, i.e., mortgage payment, taxes, insurance, and maintenance cost?

\[
\begin{align*}
N & = \\
I\% & = \\
PV & = \\
PMT & = \\
FV & = \\
P/Y & = \\
C/Y & = \\
PMT & : END
\end{align*}
\]
3. (10 points) **YOU MUST SHOW ALL FORMULAS AND CALCULATOR COMMANDS THAT YOU USE!**

Consider the following system of equations.

\[
\begin{align*}
  x + 4y - 2z &= 6 \\
  -2x - 3y &= -8 \\
  x + 4y + 5z &= -1
\end{align*}
\]

(a) Write the corresponding augmented matrix and use Gauss-Jordan Elimination method to transform the matrix to the diagonal form (if possible.) **Write the calculator command that you use.**

![Augmented Matrix](image1)  
![Diagonal Form](image2)

(b) Find all solutions.
\[
\begin{align*}
  x &= \\
  y &= \\
  z &= 
\end{align*}
\]

4. (10 points) **YOU MUST SHOW ALL FORMULAS THAT YOU USE!**

You are to form a 4-digit number from the digits 1, 2, 3, 4, 6, 8, and 9, using each at most once.

(a) Find \( n(S) \).

(b) How many of these numbers are greater than 6000?

(c) What is the probability that a randomly chosen number is greater than 6000?
5. (10 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!** (Round your answer to 4 decimal places.)

(a) 45 percent of all undergraduates at a university are chemistry majors. A random sample of 22 students were chosen. Find the probability that exactly 12 are chemistry majors.

(b) Find the value of $z$ for which the area of the shaded region under the standard normal curve is .32.

(c) Suppose that the lifetimes of a certain light bulb are normally distributed with $\mu = 5200$ hours and $\sigma = 350$. Find the probability that a light bulb will burn out in less than 5050 hours.
6. (15 points) For a certain group of states, it was observed that 60% of the Democratic governors were succeeded by Democrats and 40% by Republicans. Also, 50% of the Republican governors were succeeded by Democrats and 50% by Republicans.

(a) Set up the 2 x 2 stochastic matrix with columns and rows labeled D, and R that describes these transitions.

\[
A = \begin{bmatrix}
D & R \\
R & D
\end{bmatrix}
\]

(b) Find the stable matrix and the stable distribution. YOU MUST SHOW ALL FORMULAS OR CALCULATOR COMMANDS THAT YOU USE! Give your answers as fractions.

Stable Matrix: \[
\begin{bmatrix}
\frac{\phantom{0}}{\phantom{0}} & \frac{\phantom{0}}{\phantom{0}} \\
\frac{\phantom{0}}{\phantom{0}} & \frac{\phantom{0}}{\phantom{0}}
\end{bmatrix}
\]

and the stable distribution \[
\begin{bmatrix}
\frac{\phantom{0}}{\phantom{0}} \\
\frac{\phantom{0}}{\phantom{0}}
\end{bmatrix}
\]

(c) In the long run, what proportion of the governors will be Democrats?

7. (10 points) Of the registered voters in a certain town, 70% are Democrats, 60% favor a school loan, and 40% are Democrats who favor a school loan. Suppose that a registered voter is selected at random from the town.

(a) What is the conditional probability that the person favors the school loan given that he or she is a Democrat?

(b) What is the conditional probability that the person is a Democrat given that he or she favors the school loan?

(c) What is the probability that the person is not a Democrat and opposes the school loan?
8. (15 points) Consider the following transition diagram.

(a) Write down the transition matrix. Note that this is an absorbing stochastic matrix.

\[
\begin{bmatrix}
\quad & \quad & \quad & \quad \\
\quad & \quad & \quad & \quad \\
\quad & \quad & \quad & \quad \\
\quad & \quad & \quad & \quad \\
\end{bmatrix}
\]

(b) Find \( R, S \) and the Fundamental matrix, \( F \). YOU MUST SHOW ALL FORMULAS THAT YOU USE! Give your answers as fractions or round your answer to two decimal places.

\[
R = \begin{bmatrix}
\quad & \quad \\
\quad & \quad \\
\quad & \quad \\
\end{bmatrix}, \quad S = \begin{bmatrix}
\quad & \quad \\
\quad & \quad \\
\end{bmatrix}, \quad F = \begin{bmatrix}
\quad & \quad \\
\end{bmatrix}
\]

(c) Find the stable matrix using \( R, S, F \). YOU MUST SHOW ALL FORMULAS THAT YOU USE! Give your answers as fractions or round your answer to two decimal places.

\[
\begin{bmatrix}
\quad & \quad & \quad & \quad \\
\quad & \quad & \quad & \quad \\
\quad & \quad & \quad & \quad \\
\quad & \quad & \quad & \quad \\
\end{bmatrix}
\]
9. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU SOLVE. (Round your answer to 2 decimal places, i.e., nearest cents.)

13,000 dollars is deposited in a savings account at 14% interest compounded quarterly. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) Find the balance after 8 years.

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<tr>
<th>N</th>
<th>I%</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
<th>P/Y</th>
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<th>PMT : END</th>
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(b) How many quarters are required for the balance to reach $15,000? Note that number of quarters are always a whole number.

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<tr>
<th>N</th>
<th>I%</th>
<th>PV</th>
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10. (15 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU SOLVE. (Round your answer to 2 decimal places, i.e., nearest cents.)

Consider a $50,228 mortgage to be paid in 27 years, at interest rate 14% compounded monthly, paid at the end of the month. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) How much is the monthly payment?

\[
\begin{align*}
N &= \\
I\% &= \\
PV &= \\
PMT &= \\
FV &= \\
P/Y &= \\
C/Y &= \\
PMT : END
\end{align*}
\]

(b) How much of the first month's payment is applied to paying off the principal?

\[
\begin{align*}
N &= \\
I\% &= \\
PV &= \\
PMT &= \\
FV &= \\
P/Y &= \\
C/Y &= \\
PMT : END
\end{align*}
\]

(c) What is the unpaid balance at the end of 13 years?

\[
\begin{align*}
N &= \\
I\% &= \\
PV &= \\
PMT &= \\
FV &= \\
P/Y &= \\
C/Y &= \\
PMT : END
\end{align*}
\]
Statement of Ethics regarding this exam
I agree to complete this exam without unauthorized assistance from any person, materials, or device.
Signature: ____________________________ Date: 10 December 2014

DIRECTIONS, Read Carefully
• This is a 120 minutes exam.
• This exam consists of 10 questions. Total 110 Points. (Additional 10-Point is extra credit).
• It is your responsibility to make sure that all 9 pages are present.
• The exam is closed book. No extra papers are allowed. If you need extra paper your instructor will give you additional paper. Return the additional paper with your exam paper and write your name.
• Only TI83-TI84 calculators are allowed.
• Turn off your cell phone or any other wireless device or put it into your bag in silent mode.
• Do as Professor Alan does it in class to receive full credit.
• Remember the ethics policy.
• Best Wishes!

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Page 1 of 9
Chapter 5
Let $A, B$ be two sets.

**Inclusion Exclusion principle** $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

**Complement Rule** $n(A) = n(U) - n(A')$.

**De Morgan’s Laws**
\[
(A \cap B)' = A' \cup B' \\
(A \cup B)' = A' \cap B'
\]

Chapter 6
Let $A, B$ be two events in a sample space.

**Inclusion Exclusion principle:** $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$.

**Complement Rule** $\Pr(A) = 1 - \Pr(A')$.

**Conditional Probability** If $\Pr(B) \neq 0$. Then $\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$.

**Product Rule** If $\Pr(B) \neq 0$, then $\Pr(A \cap B) = \Pr(B) \Pr(A \mid B)$.

**Independence** $A$ and $B$ are independent if $\Pr(A \cap B) = \Pr(A) \Pr(B)$.

Chapter 7
**Probability Distribution**
Let $X$ be a random variable with numerical outcomes $x_1, x_2, \ldots, x_N$ with
\[\Pr(X = x_1) = p_1, \Pr(X = x_2) = p_2, \ldots, \Pr(X = x_N) = p_N.\]
Then
\[
\mu = E(X) = x_1 p_1 + x_2 p_2 + \cdots + x_N p_N.
\]
Variance $= \sigma^2 = (x_1 - \mu)^2 p_1 + (x_2 - \mu)^2 p_2 + \cdots + (x_N - \mu)^2 p_N$.

**Binomial Distribution** Let $X$ be a binomial random variable with parameters $n$ and $p$, then
\[
\binom{n}{k} = \frac{n!}{k!(n-k)!}.
\]
**Binomial Probability:** $\Pr(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$.

$E(X) = \mu = np$, $\sigma = \sqrt{np(1-p)}$.

Chapter 10
**Compounded Interest** Present value $P$, future value $F$, interest compounded at a rate $i$ per period
\[F = (1 + i)^n P \text{ and } P = \frac{F}{(1 + i)^n}\]

**Simple Interest** $r$ is the annual percentage rate and $n$ is the number of years, then
\[\text{Interest} = nrP, \text{ and } F = P + nrP = (1 + nr)P\]
1. (10 points) Consider the following matrices.

\[ A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \\ 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 2 \\ -3 & 0 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 2 & 4 \end{bmatrix} \]

Perform the indicated matrix calculations (if possible.) If it is not possible explain why it is not possible.

(a) \[ A + C = \]

(b) \[ A \times B = \]

(c) \[ B \times A = \]

2. (5 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU SOLVE. (Round your answer to 2 decimal places, i.e., nearest cents.)

You are considering the purchase of a condominium to use as a rental property. You estimate that you can rent the condominium for $1760 per month and that taxes, insurance, and maintenance costs will run about $240 per month.

If interest rates are 11% compounded monthly, and you plan a 29-year mortgage. How much money can you borrow and still have the rental income cover all the monthly expenses, i.e., mortgage payment, taxes, insurance, and maintenance coast?

\[
\begin{array}{l}
N = \\
I\% = \\
PV = \\
PMT = \\
FV = \\
P/Y = \\
C/Y = \\
PMT : \text{ END} \\
\end{array}
\]
3. (10 points) **YOU MUST SHOW ALL FORMULAS AND CALCULATOR COMMANDS THAT YOU USE!**

Consider the following system of equations.

\[
\begin{align*}
2x - 3y &= 9 \\
-x + 4y + 2z &= -8 \\
-2x + 3y - 2z &= -11
\end{align*}
\]

(a) Write the corresponding augmented matrix and use Gauss-Jordan Elimination method to transform the matrix to the diagonal form (if possible.) Write the calculator command that you use.

```
Augmented Matrix: 

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Calculator Command

Diagonal Form: 

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```

(b) Find all solutions.

\[x = \quad \]

\[y = \quad \]

\[z = \quad \]

4. (10 points) **YOU MUST SHOW ALL FORMULAS THAT YOU USE!**

You are to form a 5-digit number from the digits 1, 2, 3, 4, 5, 6, 8, and 9, using each at most once.

(a) Find \( n(S) \).

(b) How many of these numbers are greater than 30000?

(c) What is the probability that a randomly chosen number is greater than 30000?
5. (10 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!** (Round your answer to 4 decimal places.)

(a) 46 percent of all undergraduates at a university are chemistry majors. A random sample of 23 students were chosen. Find the probability that exactly 14 are chemistry majors.

(b) Find the value of $z$ for which the area of the shaded region under the standard normal curve is .34.

(c) Suppose that the lifetimes of a certain light bulb are normally distributed with $\mu = 5250$ hours and $\sigma = 400$. Find the probability that a light bulb will burn out in less than 5100 hours.
6. (15 points) For a certain group of states, it was observed that 70% of the Democratic governors were succeeded by Democrats and 30% by Republicans. Also, 20% of the Republican governors were succeeded by Democrats and 80% by Republicans.

(a) Set up the 2 x 2 stochastic matrix with columns and rows labeled D, and R that describes these transitions.

\[
\begin{pmatrix}
D & R \\
\hline
D & \_ \_ \\
R & \_ \_ \\
\end{pmatrix}
\]

(b) Find the stable matrix and the stable distribution. YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! Give your answers as fractions.

Stable Matrix: \[
\begin{pmatrix}
\_ \_ \\
\_ \_ \\
\end{pmatrix}
\] and the stable distribution \[
\begin{pmatrix}
\_ \\
\_ \\
\end{pmatrix}
\]

(c) In the long run, what proportion of the governors will be Democrats?

7. (10 points) Of the registered voters in a certain town, 60% are Democrats, 60% favor a school loan, and 40% are Democrats who favor a school loan. Suppose that a registered voter is selected at random from the town.

(a) What is the conditional probability that the person favors the school loan given that he or she is a Democrat?

(b) What is the conditional probability that the person is a Democrat given that he or she favors the school loan?

(c) What is the probability that the person is not a Democrat and opposes the school loan?
8. (15 points) Consider the following transition diagram.

(a) Write down the transition matrix. Note that this is an absorbing stochastic matrix.

\[
\begin{bmatrix}
0.5 & 0.3 & 0.4 & 0.1 \\
0.1 & 0.3 & 0.4 & 0.3 \\
0.1 & 0.3 & 0.3 & 0.1 \\
1 & 1 & 1 & 1
\end{bmatrix}
\]

(b) Find R, S and the Fundamental matrix, F. YOU MUST SHOW ALL FORMULAS THAT YOU USE! Give your answers as fractions or round your answer to two decimal places

\[
R = \begin{bmatrix}
\_ & \_ \\
\_ & \_ \\
\_ & \_ \\
\_ & \_
\end{bmatrix},
S = \begin{bmatrix}
\_ & \_ \\
\_ & \_ \\
\_ & \_ \\
\_ & \_
\end{bmatrix},
F = \begin{bmatrix}
\_ & \_ \\
\_ & \_ \\
\_ & \_ \\
\_ & \_
\end{bmatrix}
\]

(c) Find the stable matrix using R, S, F. YOU MUST SHOW ALL FORMULAS THAT YOU USE! Give your answers as fractions or round your answer to two decimal places

\[
\begin{bmatrix}
\_ & \_ & \_ & \_ \\
\_ & \_ & \_ & \_ \\
\_ & \_ & \_ & \_ \\
\_ & \_ & \_ & \_
\end{bmatrix}
\]
9. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU SOLVE. (Round your answer to 2 decimal places, i.e., nearest cents.)

14,000 dollars is deposited in a savings account at 16% interest compounded quarterly. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) Find the balance after 9 years.

N = 
I% = 
PV = 
PMT = 
FV = 
P/Y = 
C/Y = 
PMT : END

(b) How many quarters are required for the balance to reach $16,000? Note that number of quarters are always a whole number.

N = 
I% = 
PV = 
PMT = 
FV = 
P/Y = 
C/Y = 
PMT : END
10. (15 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE! IF YOU USE TVM SOLVER, ENTER THE NUMBERS YOU HAVE USED IN THE TABLE GIVEN BELOW. ALSO STATE CLEARLY WHICH ENTRY DID YOU SOLVE.** (Round your answer to 2 decimal places, i.e., nearest cents.)

Consider a $55,593 mortgage to be paid in 28 years, at interest rate 16% compounded monthly, paid at the end of the month. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) How much is the monthly payment?

<table>
<thead>
<tr>
<th>N</th>
<th>I%</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
<th>P/Y</th>
<th>C/Y</th>
<th>PMT END</th>
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</table>

(b) How much of the first month’s payment is applied to paying off the principal?

<table>
<thead>
<tr>
<th>N</th>
<th>I%</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
<th>P/Y</th>
<th>C/Y</th>
<th>PMT END</th>
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</table>

(c) What is the unpaid balance at the end of 14 years?

<table>
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<tr>
<th>N</th>
<th>I%</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
<th>P/Y</th>
<th>C/Y</th>
<th>PMT END</th>
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</table>
Name: ____________________________  MAT 183 U001  Final Exam

- TI 83's and 84's are the only calculators permitted. Put away all other electronic devices, including cell phones.
- SHOW ALL WORK. Partial credit will be given.
- CIRCLE YOUR ANSWERS.

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</table>
1) [10 points] Making a large decorative plate for retail sale takes \( \frac{1}{2} \) hour molding time, 3 hours oven time, and 4 hours painting time. Vases require 1 hour molding time, 2.5 hours oven time, and 3 hours painting time. Bowls require 1 hour molding time, 2 hours oven time, and 2 hours painting time. We intend to produce \( x \) plates, \( y \) vases, and \( z \) bowls.

   a. [3 points] Write three equations expressing molding time \( (m) \), oven time \( (v) \), and painting time \( (p) \) needed in terms of \( x \), \( y \), and \( z \).

   b. [3 points] Write the system as a matrix equation.

   c. [4 points] Determine the number of hours each of molding time, oven time, and painting time to produce 100 plates, 300 vases, and 200 bowls.
2) [8 points] A simplified economy consists of the two sectors Transportation and Energy. For each $1 worth of output, the transportation sector requires $0.25 worth of input from the transportation sector and $0.20 of input from the energy sector. For each $1 worth of output, the energy sector requires $0.30 from the transportation sector and $0.15 from the energy sector.

a. [2 points] Give the input-output matrix $A$ for this economy.

b. [3 points] Determine the matrix $(I - A)^{-1}$. Round entries to two decimal places.

c. [3 points] At what level of output should each sector produce to meet a demand for $5 billion worth of transportation and $3 billion worth of energy?
3) [12 points] Students at Gotham College regularly use the two browsers Internet Explorer and Firefox. Of the students who use Internet Explorer on a certain day, 70% use Internet Explorer the next day and 30% use Firefox. Of the students who use Firefox on a certain day, 60% use Internet Explorer the next day and 40% use Firefox. Suppose that during the first day of a semester 50% of the students use each browser.

a. [2 points] Set up the stochastic matrix displaying these transitions.

b. [2 points] What is the initial distribution matrix?

c. [3 points] What percent of the students will use Internet Explorer two days later?

d. [3 points] What is the stable distribution for the stochastic matrix in part a?

e. [2 points] Explain in a sentence the meaning of the number in position $a_{11}$ of the stable distribution from part d.
4) [10 points] College math departments have been rapidly establishing their own computer labs. Of the departments with no labs, each year 10% set up labs using Apple computers (A), 30% set up labs using PC's (P), and the remainder do not set up labs that year (N). Once a lab has been established with a certain type of computer, the lab is never abandoned and the brand of computer is never changed.

   a. [2 points] Set up the absorbing stochastic matrix with states A, P, N that describes this transition.

   b. [4 points] Find the following:

      i. \( R = \)

      ii. \( S = \)

      iii. \( F = \)

      iv. The stable matrix

   c. [2 points] In the long run, what percent of the math departments will have Apple computer labs?

   d. [2 points] What is the expected number of years required for a math department to decide to set up its own computer lab if it currently does not have a lab?
5) [3 points] An Internet company is considering three candidates for CEO, five candidates for CFO, and four candidates for marketing director. In how many different ways can these positions be filled?

6) [3 points] The 30 students in a math class consist of 10 science majors and 20 humanities majors. In how many ways can a group of 6 students be selected so that 3 students from each type of major are in the group?

7) [4 points] Emilio tosses a fair coin three times and observes the sequence of heads and tails.
   a. [2 points] What is the sample space for this experiment?
   b. [2 points] What is the probability that he gets at least one head and at least one tail?
8) [4 points] A finite mathematics class of 50 students is composed of freshmen and sophomores according to the following table. A student from the class is selected at random.

<table>
<thead>
<tr>
<th></th>
<th>Freshmen</th>
<th>Sophomores</th>
</tr>
</thead>
<tbody>
<tr>
<td>Females</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>Males</td>
<td>14</td>
<td>16</td>
</tr>
</tbody>
</table>

a. [2 points] What is the probability that the selected student is either female or a sophomore?

b. [2 points] What is the probability that the selected student is male, given that the student is a freshman?

9) [4 points] An urn contains three red balls and four white balls. A sample of three balls is selected at random and the number of red balls observed. Determine the probability distribution.

<table>
<thead>
<tr>
<th>Number of Red Balls</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
10) [8 points] Using life insurance tables, a retired couple determines that the probability of living 5 more years is .9 for the man and .95 for the woman. They decide to take out a life insurance policy that will pay $10,000 if either one dies during the next 5 years and $15,000 if both die during that time.

a. [6 points] Determine the probability distribution of the possible outcomes.

<table>
<thead>
<tr>
<th>Policy pays out:</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0</td>
<td></td>
</tr>
<tr>
<td>$10,000</td>
<td></td>
</tr>
<tr>
<td>$15,000</td>
<td></td>
</tr>
</tbody>
</table>

b. [2 points] How much should they be willing to pay for this policy? Assume that their life spans are independent events.
11) [5 points] Thirty percent of all cars crossing a toll bridge have a commuter sticker.

   a. [3 points] What is the probability that among 10 randomly selected cars waiting to cross the bridge at least 1 has a commuter sticker?

   b. [2 points] If 250 cars cross the bridge, what is the expected number of cars that have commuter stickers?

12) [6 points] Assume that SAT verbal scores for a first year class at a university are normally distributed with mean 520 and standard deviation 75.

   a. [3 points] The top 10% of the students are placed into the honors program for English. What is the lowest score for admittance into the honors program?

   b. [3 points] What is the range of the middle 90% of the SAT verbal scores at this university?
13) [3 points] Mr. West wishes to purchase a condominium for $240,000 cash upon his retirement 10 years from now. How much should he deposit at the end of each month into an annuity paying 6% interest compounded monthly in order to accumulate the required savings?

14) [3 points] Mrs. East would like to deposit a lump sum of money into a savings account paying 6% interest compounded monthly so that she will have $50,000 in 10 years. How much should she deposit now?

15) [4 points] Moira takes out a 30-year mortgage at 6.8% interest compounded monthly. What will the remaining balance be after 15 years?
16) [4 points] Graham opens an account with $300 and then deposits an additional $50 at the end of each month. What is the balance after 4 years if the money earns 6% interest compounded monthly?

17) [4 points] A three-year loan for $4000 is advertised at 10% using the add-on method. What is the monthly payment?

18) [5 points] Consider a 30-year mortgage of $500,000 at 7.8% interest compounded monthly where the loan is interest-only for ten years.
   a. [2 points] What is the monthly payment during the first ten years?
   b. [3 points] What is the monthly payment during the last twenty years?