**Statement of Ethics regarding this exam**

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: __________________________ Date: 6 May 2013

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**VERSION A**
### Chapter 5

Let $A$, $B$ be two sets.

**Inclusion Exclusion principle**

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

**Complement Rule**

$$n(A) = 1 - n(A').$$

**De Morgan’s Laws** Let $S$ and $T$ be sets. Then

$$(S \cap T)' = S' \cup T'$$

$$(S \cup T)' = S' \cap T'.$$

---

#### Table 1: Areas under the standard normal curve

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![Standard Normal Distribution](image-url)
Chapter 6
Let $A, B$ be two events in a sample space.

**Inclusion Exclusion principle**
\[
\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B).
\]

**Complement Rule**
\[
\Pr(A) = 1 - \Pr(A').
\]

**Conditional Probability** If $\Pr(B) \neq 0$. Then
\[
\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}.
\]

**Product Rule** If $\Pr(B) \neq 0$, then
\[
\Pr(A \cap B) = \Pr(B) \Pr(A \mid B).
\]

**Independence** $A$ and $B$ are independent if
\[
\Pr(A \cap B) = \Pr(A) \Pr(B).
\]

Chapter 7

**Probability Distribution**
Let $X$ be a random variable with numerical outcomes $x_1, x_2, \ldots, x_N$ with
\[
\Pr(X = x_1) = p_1, \Pr(X = x_2) = p_2, \ldots, \Pr(X = x_N) = p_N.
\]
Then
\[
\mu = E(X) = x_1p_1 + x_2p_2 + \cdots + x_Np_N.
\]
Variance $\sigma^2 = (x_1 - \mu)^2p_1 + (x_2 - \mu)^2p_2 + \cdots + (x_N - \mu)^2p_N$.

**Binomial Distribution**

$X$ is a binomial random variable with parameters $n$ and $p$, then

**Binomial Coefficient**
\[
\binom{n}{k} = \frac{n!}{k!(n-k)!}.
\]

**Binomial Probability**
\[
\Pr(X = k) = \binom{n}{k}p^k(1-p)^{n-k}.
\]
\[
E(X) = \mu = np, \quad \sigma = \sqrt{np(1-p)}.
\]
\[
\Pr(r \leq X \leq s) = \text{sum(seq(binompdf(n,p,X),X,r,s))}.
\]
Chebychev’s Inequality: Suppose that a probability distribution with numerical outcomes has expected value $\mu$ and standard deviation $\sigma$. Then the probability that a randomly chosen outcome lies between $\mu - c$ and $\mu + c$ is at least

$$1 - \left(\frac{c}{\sigma}\right)^2.$$ 

Chapter 10
Present value $P$, future value $F$, interest compounded at a rate $i$ per period

$$F = (1 + i)^n P$$

Increasing Annuity $n$ payments $S$ each, interest compounded at a rate $i$ per period. Future value $F$

$$F = \frac{(1 + i)^n - 1}{i} R$$

Decreasing Annuity $n$ payments $S$ each, interest compounded at a rate $i$ per period. Present value $P$

$$P = \frac{(1 + i)^n - 1}{i(1 + i)^n} R$$

1. (10 points) Let

$$A = \begin{bmatrix} 4 & -2 \\ -5 & 7 \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 & 1 \\ 5 & -1 \end{bmatrix}$$

Find the following.

(a) $A + B = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix}$

(b) $AB = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix}$

(c) $B - A = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix}$
2. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Each of four people randomly chooses one of four calculus sections to take (A, B, C, or D).
(Round your answers to four decimal places.)
(a) What is the probability that they all choose the same one?
(b) What is the probability that they each choose a different section?

3. (10 points) After applying the Gauss-Jordan elimination, a system equations have been reduced to the following.
(a) Find all solutions (if there is any), of the following system.
\[
\begin{bmatrix}
1 & 4 & 0 & 5 \\
0 & 0 & 1 & 2 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
\[x = \frac{5}{4}, \quad y = \frac{-2}{4}, \quad z = \frac{-5}{4}\]

(b) Find all solutions (if there is any), of the following system.
\[
\begin{bmatrix}
1 & 4 & 0 & 5 \\
0 & 0 & 1 & 2 \\
0 & 0 & 0 & 9
\end{bmatrix}
\]
\[x = \frac{5}{4}, \quad y = \frac{-2}{4}, \quad z = \frac{-9}{4}\]
4. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Consider the following system of equations.
\[
\begin{align*}
3x + y + z &= 9 \\
5x - y + z &= 5 \\
7x - 2y + 3z &= 3
\end{align*}
\]
(a) Write the corresponding augmented matrix.
\[
\begin{bmatrix}
\text{3} & \text{1} & \text{1} & | & \text{9} \\
\text{5} & \text{-1} & \text{1} & | & \text{5} \\
\text{7} & \text{-2} & \text{3} & | & \text{3}
\end{bmatrix}
\]
(b) Transform the matrix to the diagonal form (if possible.)
\[
\begin{bmatrix}
\text{1} & \text{0} & \text{0} & | & \text{1} \\
\text{0} & \text{1} & \text{0} & | & \text{2} \\
\text{0} & \text{0} & \text{1} & | & \text{3}
\end{bmatrix}
\]
(c) Find all solution.
\[x = \text{______}, \quad y = \text{______}, \quad z = \text{______}\]

5. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
A plumbing-supplies manufacturer produces faucet washers that are packaged in boxes of 400. Quality control studies have shown that 7% of the washers are defective. Approximate the probability using normal approximation that more than 30 of the washers in a single box are defective? Check the rule of thumb. (Round your answer to 4 decimal places.)
6. **(10 points)** YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Suppose that the lifetimes of a certain light bulb are normally distributed with $\mu = 2000$ hours and $\sigma = 400$. Find the probability that a light bulb will burn out in less than 1800 hours. (Round your answers to four decimal places.)

7. **(10 points)** YOU MUST SHOW YOUR WORK
A universal set $U$ has three subsets, A, B, and C. The numbers of elements in various subsets of $U$ are given below. Find $n(A \cup B^c)$? Show the number of elements in each fundamental regions on the venn diagram.

\[
\begin{align*}
n(U) & = 300 & n(A \cap B) & = 60 \\
n(A) & = 120 & n(B \cap C) & = 50 \\
n(B) & = 130 & n(A \cap C) & = 70 \\
n(C) & = 110 & n(A \cap B \cap C) & = 30
\end{align*}
\]

8. **(10 points)** YOU MUST SHOW ALL FORMULAS THAT YOU USE!
A code is formed by selecting four letters from the set $L = \{A, B, C, D, E, F, G, H, I, J\}$, without replacement. For example, JEFG and HGFD are two possible codes, but AAFD is not a possible code. How many such codes will contain the letter J? (Round your answers to four decimal places.)
9. (15 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Eleven thousand dollars is deposited in a savings account at 6% interest compounded monthly. (Round your answer to 2 decimal places, i.e., nearest cents.)
(a) Find the balance after 14 years.

(b) Find the amount of interest earned during that time.

(c) Find the effective rate.

10. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
The table below shows the number of advanced degrees (in thousands) earned in the U.S. during a recent year by gender and type of degree. Find the probability of a person selected at random who received an advanced degree with the following restrictions. (Round your answers to four decimal places.)

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<th>Bachelor's</th>
<th>Master's</th>
<th>Doctor's</th>
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<tr>
<td>Female</td>
<td>775</td>
<td>301</td>
<td>22</td>
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(a) The person is female, given that they received a bachelor's degree.

(b) The person has received a doctor's degree, given that they are male.
11. (25 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COM-
MANDS THAT YOU USE!
Consider a $70,000 mortgage, paid in 30 year at interest rate 7% compounded monthly,
paid at the end of the month. (Round your answer to 2 decimal places, i.e., nearest
cents.)
(a) How much is the monthly payment?

(b) How much of the first month's payment is applied to paying off the principal?

(c) What is the unpaid balance at the end of 25 years?

(d) How much of the principal is repaid during the 26th year?

(e) How much interest is paid during the 301st month?

12. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COM-
MANDS THAT YOU USE!
A single die is tossed 20 times. Find the probability that exactly 9 of the tosses show either
"four" or "five", (Round your answer to 4 decimal places.)
13. (20 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!**

Suppose that the following data were obtained from the records of a certain two-year college. Of those who were freshmen (F) during a particular year, 60% became sophomores (S) the next year and 40% dropped out (D). Of those who were sophomores during a particular year, 75% graduated (G) by the next year and 25% dropped out. (Round your answers to four decimal places.)

(a) Set up the absorbing stochastic matrix with states D, G, F, S that describes this transition.

\[
\begin{bmatrix}
D & G & F & S \\
0 & 0 & 0.60 & 0.40 \\
0 & 0.75 & 0 & 0.25 \\
1 & 0 & 0 & 0
\end{bmatrix}
\]

(b) Find R, S and Fundamental matrix.

\[
R = \begin{bmatrix} D \\ G \\ F \end{bmatrix}, \quad S = \begin{bmatrix} F \\ S \end{bmatrix}
\]

\[
F = \begin{bmatrix} D \\ G \\ F \\ S \end{bmatrix}
\]

(c) Find the stable matrix.

\[
\begin{bmatrix}
D & G \\
F & S \\
0 & 0 \\
1 & 0
\end{bmatrix}
\]

(d) If there are 900 freshmen and 600 sophomores at the beginning of the year, how many of them will drop out at the end of the year?
Elements of Modern Mathematics  MAT 183
SPRING 2013
FINAL Exam

NAME LABEL

Statement of Ethics regarding this exam
I agree to complete this exam without unauthorized assistance from any person, materials, or device.
Signature: __________________ Date: 6 May 2013

DIRECTIONS, Read Carefully

• This exam consists of 13 questions worth 160 points. Answer all questions.

• It is your responsibility to make sure that all 10 pages are present.

• The exam is closed book. No extra papers are allowed. If you need extra paper your instructor will give you additional paper. Return the additional paper with your exam paper and write your name.

• Only TI83-TI84 calculators are allowed.

• Turn off your cell phone or any other wireless device or put it into your bag in silent mode.

• Follow the instructions to receive full credit. Just answers or irrelevant calculations receive no credit.

• Remember the ethics policy.

• Best Wishes!

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VERSION B

Page 1 of 10
Chapter 5

Let $A$, $B$ be two sets.

Inclusion Exclusion principle

\[ n(A \cup B) = n(A) + n(B) - n(A \cap B). \]

Complement Rule

\[ n(A) = 1 - n(A'). \]

De Morgan’s Laws

Let $S$ and $T$ be sets. Then

\[
(S \cap T)' = S' \cup T'
\]

\[
(S \cup T)' = S' \cap T'
\]
Chapter 6
Let $A$, $B$ be two events in a sample space.

Inclusion Exclusion principle

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B).$$

Complement Rule

$$\Pr(A) = 1 - \Pr(A').$$

Conditional Probability If $\Pr(B) \neq 0$. Then

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}.$$

Product Rule If $\Pr(B) \neq 0$, then

$$\Pr(A \cap B) = \Pr(B) \Pr(A \mid B).$$

Independence $A$ and $B$ are independent if

$$\Pr(A \cap B) = \Pr(A) \Pr(B).$$

Chapter 7
Probability Distribution
Let $X$ be a random variable with numerical outcomes $x_1, x_2, \ldots, x_N$ with

$$\Pr(X = x_1) = p_1, \Pr(X = x_2) = p_2, \ldots, \Pr(X = x_N) = p_N.$$ Then

$$\mu = E(X) = x_1 p_1 + x_2 p_2 + \cdots + x_N p_N.$$ 

Variance $\sigma^2 = (x_1 - \mu)^2 p_1 + (x_2 - \mu)^2 p_2 + \cdots + (x_N - \mu)^2 p_N.$

Binomial Distribution
$X$ is a binomial random variable with parameters $n$ and $p$, then

Binomial Coefficient

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}. $$

Binomial Probability

$$\Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}. $$

$$E(X) = \mu = np, \quad \sigma = \sqrt{np(1-p)}. $$

$$\Pr(r \leq X \leq s) = \text{sum(seq(binompdf(n,p,X), X,r,s))}. $$
Chebychev’s Inequality: Suppose that a probability distribution with numerical outcomes has expected value $\mu$ and standard deviation $\sigma$. Then the probability that a randomly chosen outcome lies between $\mu - c$ and $\mu + c$ is at least

$$1 - \left(\frac{c}{\sigma}\right)^2.$$  

Chapter 10

Present value $P$, future value $F$, interest compounded at a rate $i$ per period

$$F = (1 + i)^n P$$

Increasing Annuity $n$ payments $\$R$ each, interest compounded at a rate $i$ per period. Future value $F$

$$F = \frac{(1 + i)^n - 1}{i} R$$

Decreasing Annuity $n$ payments $\$R$ each, interest compounded at a rate $i$ per period. Present value $P$

$$P = \frac{(1 + i)^n - 1}{i(1 + i)^n} R$$

1. (10 points) Let

$$A = \begin{bmatrix} 9 & -3 \\ -4 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 7 \\ 4 & -2 \end{bmatrix}$$

Find the following.

(a) $A + B = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix}$

(b) $AB = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix}$

(c) $B - A = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix}$
2. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COM­MANDS THAT YOU USE!
Each of five people randomly chooses one of five calculus sections to take (A, B, C, D, or E). (Round your answers to four decimal places.)
(a) What is the probability that they all choose the same one?

(b) What is the probability that they each choose a different section?

3. (10 points) After applying the Gauss-Jordan elimination, a system of equations have been reduced to the following.
(a) Find all solutions (if there is any), of the following system.

\[
\begin{bmatrix}
1 & 3 & 0 & 4 \\
0 & 0 & 1 & 5 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[x = \underline{\phantom{0000}}, \quad y = \underline{\phantom{0000}}, \quad z = \underline{\phantom{0000}}\]

(b) Find all solutions (if there is any), of the following system.

\[
\begin{bmatrix}
1 & 3 & 0 & 4 \\
0 & 0 & 1 & 5 \\
0 & 0 & 0 & 6
\end{bmatrix}
\]

\[x = \underline{\phantom{0000}}, \quad y = \underline{\phantom{0000}}, \quad z = \underline{\phantom{0000}}\]
4. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Consider the following system of equations.
\[
\begin{align*}
2x + 3y + 2z &= 3 \\
3x + 2y - 2z &= 2 \\
x - y - 3z &= 1
\end{align*}
\]
(a) Write the corresponding augmented matrix.
\[
\begin{bmatrix}
\_ & \_ & \_ & | & \_ \\
\_ & \_ & \_ & | & \_ \\
\_ & \_ & \_ & | & \_
\end{bmatrix}
\]
(b) Transform the matrix to the diagonal form (if possible.)
\[
\begin{bmatrix}
\_ & \_ & \_ & | & \_ \\
\_ & \_ & \_ & | & \_ \\
\_ & \_ & \_ & | & \_
\end{bmatrix}
\]
(c) Find all solution.
\[x = \_ , \ y = \_ , \ z = \_
\]

5. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
A plumbing-supplies manufacturer produces faucet washers that are packaged in boxes of 500. Quality control studies have shown that 8% of the washers are defective. Approximate the probability using normal approximation that more than 40 of the washers in a single box are defective? Check the rule of thumb. (Round your answer to 4 decimal places.)
6. (10 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!**
Suppose that the lifetimes of a certain light bulb are normally distributed with $\mu = 3000$ hours and $\sigma = 500$. Find the probability that a light bulb will burn out in less than 2700 hours. (Round your answers to four decimal places.)

7. (10 points) **YOU MUST SHOW YOUR WORK**
A universal set $U$ has three subsets, $A$, $B$, and $C$. The numbers of elements in various subsets of $U$ are given below. Find $n(A \cup B')$? Show the number of elements in each fundamental regions on the venn diagram.

\[ n(U) = 500 \quad n(A \cap B) = 80 \]
\[ n(A) = 150 \quad n(B \cap C) = 60 \]
\[ n(B) = 140 \quad n(A \cap C) = 70 \]
\[ n(C) = 120 \quad n(A \cap B \cap C) = 40 \]

8. (10 points) **YOU MUST SHOW ALL FORMULAS THAT YOU USE!**
A code is formed by selecting four letters from the set $L = \{A, B, C, D, E, F, G, H, I, J\}$, without replacement. For example, JEAFG and HGBFD are two possible codes, but CAAFD is not a possible code. How many such codes will contain the letter $J$? (Round your answers to four decimal places.)
9. (15 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Twelve thousand dollars is deposited in a savings account at 7% interest compounded monthly. (Round your answer to 2 decimal places, i.e., nearest cents.)
(a) Find the balance after 15 years.
(b) Find the amount of interest earned during that time.
(c) Find the effective rate.

10. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
The table below shows the number of advanced degrees (in thousands) earned in the U.S. during a recent year by gender and type of degree. Find the probability of a person selected at random who received an advanced degree with the following restrictions. (Round your answers to four decimal places.)

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<thead>
<tr>
<th>Bachelor's</th>
<th>Master's</th>
<th>Doctor's</th>
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<tr>
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<td>211</td>
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<tr>
<td>Female</td>
<td>775</td>
<td>301</td>
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</tbody>
</table>

(a) The person is female, given that they received a master's degree.
(b) The person has received a bachelor's degree, given that they are male.
11. (25 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Consider a $80,000 mortgage, paid in 30 years at interest rate 5% compounded monthly, paid at the end of the month. (Round your answer to 2 decimal places, i.e., nearest cents.)
(a) How much is the monthly payment?

(b) How much of the first month's payment is applied to paying off the principal?

(c) What is the unpaid balance at the end of 25 years?

(d) How much of the principal is repaid during the 26th year?

(e) How much interest is paid during the 301st month?

12. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
A single die is tossed 21 times. Find the probability that exactly 10 of the tosses show either "four" or "five", (Round your answer to 4 decimal places.)
13. (20 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COM-
MANDS THAT YOU USE!
Suppose that the following data were obtained from the records of a certain two-year college. Of those who were freshmen (F) during a particular year, 65% became sophomores (S) the next year and 35% dropped out (D). Of those who were sophomores during a particular year, 70% graduated (G) by the next year and 30% dropped out. (Round your answers to four decimal places.)

(a) Set up the absorbing stochastic matrix with states D, G, F, S that describes this transition.

(b) Find R, S and Fundamental matrix.

(c) Find the stable matrix.

(d) If there are 800 freshmen and 500 sophomores at the beginning of the year, how many of them will drop out at the end of the year?
Statement of Ethics regarding this exam
I agree to complete this exam without unauthorized assistance from any person, materials, or device.
Signature: __________________________ Date: 6 May 2013

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VERSION C

Page 1 of 10
Chapter 5

Let $A, B$ be two sets.

Inclusion Exclusion principle

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

Complement Rule

$$n(A) = 1 - n(A').$$

De Morgan’s Laws Let $S$ and $T$ be sets. Then

$$(S \cap T)' = S' \cup T'$$

$$(S' \cup T)' = S \cap T'$$
Chapter 6
Let $A, B$ be two events in a sample space.

Inclusion Exclusion principle

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B).$$

Complement Rule

$$\Pr(A) = 1 - \Pr(A').$$

Conditional Probability  If $\Pr(B) \neq 0$. Then

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}.$$

Product Rule  If $\Pr(B) \neq 0$, then

$$\Pr(A \cap B) = \Pr(B) \Pr(A \mid B).$$

Independence  $A$ and $B$ are independent if

$$\Pr(A \cap B) = \Pr(A) \Pr(B).$$

Chapter 7

Probability Distribution
Let $X$ be a random variable with numerical outcomes $x_1, x_2, \ldots, x_N$ with

$$\Pr(X = x_1) = p_1, \Pr(X = x_2) = p_2, \ldots, \Pr(X = x_N) = p_N.$$

Then

$$\mu = E(X) = x_1p_1 + x_2p_2 + \cdots + x_Np_N.$$  

Variance  $\sigma^2 = (x_1 - \mu)^2p_1 + (x_2 - \mu)^2p_2 + \cdots + (x_N - \mu)^2p_N$.

Binomial Distribution
$X$ is a binomial random variable with parameters $n$ and $p$, then

Binomial Coefficient

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

Binomial Probability

$$\Pr(X = k) = \binom{n}{k}p^k(1-p)^{n-k}.$$

$$E(X) = \mu = np, \quad \sigma = \sqrt{np(1-p)}.$$  

$$\Pr(r \leq X \leq s) = \text{sum(seq(binompdf(n,p,X),X,r,s))}.$$
Chebychev's Inequality: Suppose that a probability distribution with numerical outcomes has expected value \( \mu \) and standard deviation \( \sigma \). Then the probability that a randomly chosen outcome lies between \( \mu - c \) and \( \mu + c \) is at least

\[
1 - \left( \frac{\sigma}{c} \right)^2.
\]

Chapter 10

Present value \( P \), future value \( F \), interest compounded at a rate \( i \) per period

\[
F = (1 + i)^n P
\]

**Increasing Annuity** \( n \) payments \( R \) each, interest compounded at a rate \( i \) per period. Future value \( F \)

\[
F = \frac{(1 + i)^n - 1}{i} R
\]

**Decreasing Annuity** \( n \) payments \( R \) each, interest compounded at a rate \( i \) per period. Present value \( P \)

\[
P = \frac{(1 + i)^n - 1}{i(1 + i)^n} R
\]

1. (10 points) Let

\[ A = \begin{bmatrix} 4 & -2 \\ -3 & 6 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 4 \\ 9 & -3 \end{bmatrix} \]

Find the following.

(a) \( A + B = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix} \)

(b) \( AB = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix} \)

(c) \( B - A = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix} \)

Page 4 of 10
2. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Each of six people randomly chooses one of six calculus sections to take (A, B, C, D, E, F). (Round your answers to four decimal places.)
(a) What is the probability that they all choose the same one?

(b) What is the probability that they each choose a different section?

3. (10 points) After applying the Gauss-Jordan elimination, a system of equations have been reduced to the following.
(a) Find all solutions (if there is any), of the following system.
\[ \begin{bmatrix} 1 & 4 & 0 & 3 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]
\[ x = \_\_\_\_, \quad y = \_\_\_\_, \quad z = \_\_\_\_ \]

(b) Find all solutions (if there is any), of the following system.
\[ \begin{bmatrix} 1 & 4 & 0 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 4 \end{bmatrix} \]
\[ x = \_\_\_\_, \quad y = \_\_\_\_, \quad z = \_\_\_\_ \]
4. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COM-MANDS THAT YOU USE!
Consider the following system of equations.
\[
\begin{align*}
5x + 4y + z &= 1 \\
4x + 5y + 2z &= 8 \\
-x + y - z &= 1
\end{align*}
\]
(a) Write the corresponding augmented matrix.
\[
\begin{bmatrix}
\_ & \_ & \_ & | & \_ \\
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\_ & \_ & \_ & | & \_ \\
\end{bmatrix}
\]
(b) Transform the matrix to the diagonal form (if possible.)
\[
\begin{bmatrix}
\_ & \_ & \_ & | & \_ \\
\_ & \_ & \_ & | & \_ \\
\_ & \_ & \_ & | & \_ \\
\end{bmatrix}
\]
(c) Find all solution.
\[
x = \_ \_ \_, \quad y = \_ \_ \_, \quad z = \_ \_ \_
\]

5. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COM-MANDS THAT YOU USE!
A plumbing-supplies manufacturer produces faucet washers that are packaged in boxes of 600. Quality control studies have shown that 7% of the washers are defective. Approximate the probability using normal approximation that more than 45 of the washers in a single box are defective? Check the rule of thumb. (Round your answer to 4 decimal places.)
6. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Suppose that the lifetimes of a certain light bulb are normally distributed with \( \mu = 2500 \) hours and \( \sigma = 400 \). Find the probability that a light bulb will burn out in less than 2600 hours. (Round your answers to four decimal places.)

7. (10 points) YOU MUST SHOW YOUR WORK
A universal set \( U \) has three subsets, \( A \), \( B \), and \( C \). The numbers of elements in various subsets of \( U \) are given below. Find \( n(A \cup B') \)? Show the number of elements in each fundamental regions on the venn diagram.

\[
\begin{align*}
n(U) &= 500 \\
n(A) &= 140 \\
n(B) &= 150 \\
n(C) &= 120 \\
n(A \cap B) &= 70 \\
n(B \cap C) &= 80 \\
n(A \cap C) &= 60 \\
n(A \cap B \cap C) &= 20
\end{align*}
\]

8. (10 points) YOU MUST SHOW ALL FORMULAS THAT YOU USE!
A code is formed by selecting six letters from the set \( L = \{A, B, C, D, E, F, G, H, I, J\} \), without replacement. For example, JEAFGH and AHGBFD are two possible codes, but ICAAFLD is not a possible code. How many such codes will contain the letter \( J \)? (Round your answers to four decimal places.)
9. (15 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!

Thirteen thousand dollars is deposited in a savings account at 8% interest compounded monthly. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) Find the balance after 16 years.

(b) Find the amount of interest earned during that time.

(c) Find the effective rate.

10. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!

The table below shows the number of advanced degrees (in thousands) earned in the U.S. during a recent year by gender and type of degree. Find the probability of a person selected at random who received an advanced degree with the following restrictions. (Round your answers to four decimal places.)

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<tr>
<th>Type</th>
<th>Bachelor's</th>
<th>Master's</th>
<th>Doctor's</th>
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<tr>
<td>Female</td>
<td>775</td>
<td>301</td>
<td>22</td>
</tr>
</tbody>
</table>

(a) The person is male, given that they received a master's degree.

(b) The person has received a bachelor's degree, given that they are female.
11. (25 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!**

Consider a $90,000 mortgage, paid in 30 years at interest rate 11% compounded monthly, paid at the end of the month. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) How much is the monthly payment?

(b) How much of the first month's payment is applied to paying off the principal?

(c) What is the unpaid balance at the end of 25 years?

(d) How much of the principal is repaid during the 26th year?

(e) How much interest is paid during the 301st month?

12. (10 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!**

A single die is tossed 22 times. Find the probability that exactly 11 of the tosses show either "four" or "five", (Round your answer to 4 decimal places.)
13. (20 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!

Suppose that the following data were obtained from the records of a certain two-year college. Of those who were freshmen (F) during a particular year, 65% became sophomores (S) the next year and 35% dropped out (D). Of those who were sophomores during a particular year, 80% graduated (G) by the next year and 20% dropped out. (Round your answers to four decimal places.)

(a) Set up the absorbing stochastic matrix with states D, G, F, S that describes this transition.

\[
\begin{bmatrix}
  \_ & \_ & \_ & \_ \\
  \_ & \_ & \_ & \_ \\
  \_ & \_ & \_ & \_ \\
  \_ & \_ & \_ & \_ \\
\end{bmatrix}
\]

(b) Find R, S and Fundamental matrix.

\[R = \begin{bmatrix} \_ & \_ \end{bmatrix}, \quad S = \begin{bmatrix} \_ & \_ \end{bmatrix}\]

\[F = \begin{bmatrix} \_ & \_ \end{bmatrix}\]

(c) Find the stable matrix.

\[
\begin{bmatrix}
  \_ & \_ & \_ & \_ \\
  \_ & \_ & \_ & \_ \\
  \_ & \_ & \_ & \_ \\
  \_ & \_ & \_ & \_ \\
\end{bmatrix}
\]

(d) If there are 700 freshmen and 600 sophomores at the beginning of the year, how many of them will drop out at the end of the year?
Elements of Modern Mathematics  
MAT 183  
SPRING 2013  
FINAL Exam

NAME LABEL

Statement of Ethics regarding this exam
I agree to complete this exam without unauthorized assistance from any person, materials, or device.
Signature: __________________________ Date: 6 May 2013

DIRECTIONS, Read Carefully

- This exam consists of 13 questions worth 160 points. Answer all questions.

- It is your responsibility to make sure that all 10 pages are present.

- The exam is closed book. No extra papers are allowed. If you need extra paper your instructor will give you additional paper. Return the additional paper with your exam paper and write your name.

- Only TI83-TI84 calculators are allowed.

- Turn off your cell phone or any other wireless device or put it into your bag in silent mode.

- Follow the instructions to receive full credit. Just answers or irrelevant calculations receive no credit.

- Remember the ethics policy.

- Best Wishes!

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VERSION D

Page 1 of 10
Chapter 5
Let A, B be two sets.

Inclusion Exclusion principle

\[ n(A \cup B) = n(A) + n(B) - n(A \cap B). \]

Complement Rule

\[ n(A) = 1 - n(A'). \]

De Morgan's Laws Let S and T be sets. Then

\[ (S \cap T)' = S' \cup T' \]
\[ (S \cup T)' = S' \cap T' \]
Chapter 6
Let $A, B$ be two events in a sample space.

Inclusion Exclusion principle

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B).$$

Complement Rule

$$\Pr(A) = 1 - \Pr(A').$$

Conditional Probability If $\Pr(B) \neq 0$. Then

$$\Pr(A | B) = \frac{\Pr(A \cap B)}{\Pr(B)}.$$

Product Rule If $\Pr(B) \neq 0$, then

$$\Pr(A \cap B) = \Pr(B) \Pr(A | B).$$

Independence $A$ and $B$ are independent if

$$\Pr(A \cap B) = \Pr(A) \Pr(B).$$

Chapter 7
Probability Distribution
Let $X$ be a random variable with numerical outcomes $x_1, x_2, \ldots, x_N$ with

$$\Pr(X = x_1) = p_1, \Pr(X = x_2) = p_2, \ldots, \Pr(X = x_N) = p_N.$$ Then

$$\mu = E(X) = x_1 p_1 + x_2 p_2 + \cdots + x_N p_N.$$ 

Variance $\sigma^2 = (x_1 - \mu)^2 p_1 + (x_2 - \mu)^2 p_2 + \cdots + (x_N - \mu)^2 p_N.$

Binomial Distribution
$X$ is a binomial random variable with parameters $n$ and $p$, then

Binomial Coefficient

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$ 

Binomial Probability

$$\Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}.$$ 

$$E(X) = \mu = np, \quad \sigma = \sqrt{np(1-p)}.$$ 

$$\Pr(r \leq X \leq s) = \text{sum(seq(binompdf(n, p, X), X, r, s)).}$$
Chebychev's Inequality: Suppose that a probability distribution with numerical outcomes has expected value \( \mu \) and standard deviation \( \sigma \). Then the probability that a randomly chosen outcome lies between \( \mu - c \) and \( \mu + c \) is at least 

\[
1 - \left( \frac{c}{\sigma} \right)^2
\]

Chapter 10
Present value \( P \), future value \( F \), interest compounded at a rate \( i \) per period

\[
F = (1 + i)^nP
\]

Increasing Annuity \( n \) payments \$R each, interest compounded at a rate \( i \) per period.
Future value \( F \)

\[
F = \frac{(1 + i)^n - 1}{i}R
\]

Decreasing Annuity \( n \) payments \$R each, interest compounded at a rate \( i \) per period.
Present value \( P \)

\[
P = \frac{(1 + i)^n - 1}{i(1 + i)^n}R
\]

1. (10 points) Let

\[
A = \begin{bmatrix} 2 & -9 \\ -5 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 4 & 7 \\ 2 & -6 \end{bmatrix}
\]

Find the following.

(a) \( A + B = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix} \)

(b) \( AB = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix} \)

(c) \( B - A = \begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix} \)
2. (10 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!**
Each of seven people randomly chooses one of seven calculus sections to take (A, B, C, D, E, F, G). (Round your answers to four decimal places.)

(a) What is the probability that they all choose the same one?

(b) What is the probability that they each choose a different section?

3. (10 points) After applying the Gauss-Jordan elimination, a system equations have been reduced to the following.

(a) Find all solutions (if there is any), of the following system.

\[
\begin{bmatrix}
1 & 2 & 0 & 9 \\
0 & 0 & 1 & 4 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[x = \_\_\_, \ y = \_\_\_, \ z = \_\_\_
\]

(b) Find all solutions (if there is any), of the following system.

\[
\begin{bmatrix}
1 & 2 & 0 & 9 \\
0 & 0 & 1 & 4 \\
0 & 0 & 0 & 11
\end{bmatrix}
\]

\[x = \_\_\_, \ y = \_\_\_, \ z = \_\_\_
\]
4. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Consider the following system of equations.
\[
\begin{align*}
2x - 3y + z &= 4 \\
3x - 2y + z &= 9 \\
x + y - z &= 1
\end{align*}
\]
(a) Write the corresponding augmented matrix.
\[
\begin{bmatrix}
\vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots \\
\end{bmatrix}
\]
(b) Transform the matrix to the diagonal form (if possible.)
\[
\begin{bmatrix}
\vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots \\
\end{bmatrix}
\]
(c) Find all solutions.
\[
x = \ldots, \quad y = \ldots, \quad z = \ldots
\]

5. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
A plumbing-supplies manufacturer produces faucet washers that are packaged in boxes of 700. Quality control studies have shown that 5% of the washers are defective. Approximate the probability using normal approximation that more than 40 of the washers in a single box are defective? Check the rule of thumb. (Round your answer to 4 decimal places.)
6. (10 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!**
Suppose that the lifetimes of a certain light bulb are normally distributed with \( \mu = 4000 \) hours and \( \sigma = 300 \). Find the probability that a light bulb will burn out in less than 4200 hours. (Round your answers to four decimal places.)

7. (10 points) **YOU MUST SHOW YOUR WORK**
A universal set \( U \) has three subsets, \( A \), \( B \), and \( C \). The numbers of elements in various subsets of \( U \) are given below. Find \( n(A \cup B') \)? Show the number of elements in each fundamental regions on the Venn diagram.

\[
\begin{align*}
n(U) &= 500 \\
n(A) &= 180 \\
n(B) &= 120 \\
n(C) &= 100 \\
n(A \cap B) &= 90 \\
n(B \cap C) &= 80 \\
n(A \cap C) &= 50 \\
n(A \cap B \cap C) &= 30
\end{align*}
\]

8. (10 points) **YOU MUST SHOW ALL FORMULAS THAT YOU USE!**
A code is formed by selecting seven letters from the set \( L = \{A, B, C, D, E, F, G, H, I, J\} \), without replacement. For example, JEAFGHB and ACHGBFD are two possible codes, but ICAAFDB is not a possible code. How many such codes will contain the letter \( J \)? (Round your answers to four decimal places.)
9. (15 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Fourteen thousand dollars is deposited in a savings account at 9% interest compounded monthly. (Round your answer to 2 decimal places, i.e., nearest cents.)
(a) Find the balance after 17 years.

(b) Find the amount of interest earned during that time.

(c) Find the effective rate.

10. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
The table below shows the number of advanced degrees (in thousands) earned in the U.S. during a recent year by gender and type of degree. Find the probability of a person selected at random who received an advanced degree with the following restrictions. (Round your answers to four decimal places.)

<table>
<thead>
<tr>
<th>Bachelor's</th>
<th>Master's</th>
<th>Doctor's</th>
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</thead>
<tbody>
<tr>
<td>Male</td>
<td>573</td>
<td>211</td>
</tr>
<tr>
<td>Female</td>
<td>775</td>
<td>301</td>
</tr>
</tbody>
</table>

(a) The person is male, given that they received a bachelor's degree.

(b) The person has received a master's degree, given that they are female.
11. (25 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Consider a $100,000 mortgage, paid in 30 year at interest rate 13% compounded monthly, paid at the end of the month. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) How much is the monthly payment?

(b) How much of the first month's payment is applied to paying off the principal?

(c) What is the unpaid balance at the end of 25 years?

(d) How much of the principal is repaid during the 26th year?

(e) How much interest is paid during the 301st month?

12. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
A single die is tossed 23 times. Find the probability that exactly 12 of the tosses show either "four" or "five", (Round your answer to 4 decimal places.)
13. (20 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COM¬
MANDS THAT YOU USE!

Suppose that the following data were obtained from the records of a certain two-year col-
lege. Of those who were freshmen (F) during a particular year, 60% became sophomores (S) the next year and 40% dropped out (D). Of those who were sophomores during a par-
ticular year, 85% graduated (G) by the next year and 15% dropped out. (Round your
answers to four decimal places.)

(a) Set up the absorbing stochastic matrix with states D, G, F, S that describes this
transition.

\[
\begin{pmatrix}
\ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots \\
\end{pmatrix}
\]

(b) Find R, S and Fundamental matrix.

\[
R = \begin{pmatrix}
\ldots & \ldots \\
\ldots & \ldots \\
\end{pmatrix}, \quad S = \begin{pmatrix}
\ldots & \ldots \\
\end{pmatrix}
\]

\[
F = \begin{pmatrix}
\ldots & \ldots \\
\ldots & \ldots \\
\end{pmatrix}
\]

(c) Find the stable matrix.

\[
\begin{pmatrix}
\ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots \\
\end{pmatrix}
\]

(d) If there are 600 freshmen and 700 sophomores at the beginning of the year, how many
of them will drop out at the end of the year?
Statement of Ethics regarding this exam
I agree to complete this exam without unauthorized assistance from any person, materials, or device.
Signature: ______________________ Date: 6 May 2013

DIRECTIONS, Read Carefully

• This exam consists of 13 questions worth 160 points. Answer all questions.

• It is your responsibility to make sure that all 10 pages are present.

• The exam is closed book. No extra papers are allowed. If you need extra paper your instructor will give you additional paper. Return the additional paper with your exam paper and write your name.

• Only TI83-TI84 calculators are allowed.

• Turn off your cell phone or any other wireless device or put it into your bag in silent mode.

• Follow the instructions to receive full credit. Just answers or irrelevant calculations receive no credit.

• Remember the ethics policy.

• Best Wishes!

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<td>20</td>
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<td><strong>Total:</strong></td>
<td><strong>160</strong></td>
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</tbody>
</table>
Chapter 5

Let $A$, $B$ be two sets.

Inclusion Exclusion principle

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

Complement Rule

$$n(A) = 1 - n(A').$$

De Morgan's Laws Let $S$ and $T$ be sets. Then

$$(S \cap T)' = S' \cup T'$$
$$(S \cup T)' = S' \cap T'$$
Chapter 6
Let $A$, $B$ be two events in a sample space.

Inclusion Exclusion principle

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B).$$

Complement Rule

$$\Pr(A) = 1 - \Pr(A').$$

Conditional Probability

If $\Pr(B) \neq 0$. Then

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}.$$

Product Rule

If $\Pr(B) \neq 0$, then

$$\Pr(A \cap B) = \Pr(B) \Pr(A \mid B).$$

Independence

$A$ and $B$ are independent if

$$\Pr(A \cap B) = \Pr(A) \Pr(B).$$

Chapter 7
Probability Distribution

Let $X$ be a random variable with numerical outcomes $x_1, x_2, \ldots, x_N$ with

$$\Pr(X = x_1) = p_1, \Pr(X = x_2) = p_2, \ldots, \Pr(X = x_N) = p_N.$$ Then

$$\mu = E(X) = x_1p_1 + x_2p_2 + \cdots + x_Np_N.$$ 

Variance

$$\sigma^2 = (x_1 - \mu)^2p_1 + (x_2 - \mu)^2p_2 + \cdots + (x_N - \mu)^2p_N.$$ 

Binomial Distribution

$X$ is a binomial random variable with parameters $n$ and $p$, then

Binomial Coefficient

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$ 

Binomial Probability

$$\Pr(X = k) = \binom{n}{k}p^k(1-p)^{n-k}.$$ 

$$E(X) = \mu = np, \quad \sigma = \sqrt{np(1-p)}.$$ 

$$\Pr(r \leq X \leq s) = \text{sum(seq(binompdf(n,p,X),X,r,s)).}$$
Chebychev's Inequality: Suppose that a probability distribution with numerical outcomes has expected value $\mu$ and standard deviation $\sigma$. Then the probability that a randomly chosen outcome lies between $\mu - c$ and $\mu + c$ is at least

$$1 - \left(\frac{\sigma}{c}\right)^2.$$
2. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Each of four people randomly chooses one of four calculus sections to take (A, B, C, or D).
(Round your answers to four decimal places.)

(a) What is the probability that they all choose the same one?

(b) What is the probability that they each choose a different section?

3. (10 points) After applying the Gauss-Jordan elimination, a system equations have been reduced to the following.

(a) Find all solutions (if there is any), of the following system.

\[
\begin{bmatrix}
1 & 4 & 0 & 5 \\
0 & 0 & 1 & 2 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[x = \ldots, \quad y = \ldots, \quad z = \ldots\]

(b) Find all solutions (if there is any), of the following system.

\[
\begin{bmatrix}
1 & 4 & 0 & 5 \\
0 & 0 & 1 & 2 \\
0 & 0 & 0 & 9 \\
\end{bmatrix}
\]

\[x = \ldots, \quad y = \ldots, \quad z = \ldots\]
4. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!

Consider the following system of equations.

\[
\begin{align*}
3x + y + z &= 9 \\
5x - y + z &= 5 \\
7x - 2y + 3z &= 3
\end{align*}
\]

(a) Write the corresponding augmented matrix.

\[
\begin{bmatrix}
\cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
\end{bmatrix}
\]

(b) Transform the matrix to the diagonal form (if possible.)

\[
\begin{bmatrix}
\cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots \\
\end{bmatrix}
\]

(c) Find all solution.

\[x = \ldots, \quad y = \ldots, \quad z = \ldots\]

5. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!

A plumbing-supplies manufacturer produces faucet washers that are packaged in boxes of 400. Quality control studies have shown that 7% of the washers are defective. Approximate the probability using normal approximation that more than 30 of the washers in a single box are defective? Check the rule of thumb. (Round your answer to 4 decimal places.)
6. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Suppose that the lifetimes of a certain light bulb are normally distributed with \( \mu = 2000 \) hours and \( \sigma = 400 \). Find the probability that a light bulb will burn out in less than 1800 hours. (Round your answers to four decimal places.)

7. (10 points) YOU MUST SHOW YOUR WORK
A universal set \( U \) has three subsets, \( A, B, \) and \( C \). The numbers of elements in various subsets of \( U \) are given below. Find \( n(A \cup B') \)? Show the number of elements in each fundamental regions on the venn diagram.

\[
\begin{align*}
n(U) &= 300 \quad n(A \cap B) = 60 \\
n(A) &= 120 \quad n(B \cap C) = 50 \\
n(B) &= 130 \quad n(A \cap C) = 70 \\
n(C) &= 110 \quad n(A \cap B \cap C) = 30
\end{align*}
\]

8. (10 points) YOU MUST SHOW ALL FORMULAS THAT YOU USE!
A code is formed by selecting four letters from the set \( L = \{A, B, C, D, E, F, G, H, I, J\} \), without replacement. For example, JEFG and HGFD are two possible codes, but AAFD is not a possible code. How many such codes will contain the letter \( J \)? (Round your answers to four decimal places.)
9. (15 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!

Eleven thousand dollars is deposited in a savings account at 6% interest compounded monthly. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) Find the balance after 14 years.

(b) Find the amount of interest earned during that time.

(c) Find the effective rate.

10. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!

The table below shows the number of advanced degrees (in thousands) earned in the U.S. during a recent year by gender and type of degree. Find the probability of a person selected at random who received an advanced degree with the following restrictions. (Round your answers to four decimal places.)

<table>
<thead>
<tr>
<th>Gender</th>
<th>Bachelor's</th>
<th>Master's</th>
<th>Doctor's</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>573</td>
<td>211</td>
<td>24</td>
</tr>
<tr>
<td>Female</td>
<td>775</td>
<td>301</td>
<td>22</td>
</tr>
</tbody>
</table>

(a) The person is female, given that they received a bachelor's degree.

(b) The person has received a doctor's degree, given that they are male.
11. (25 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Consider a $70,000 mortgage, paid in 30 years at interest rate 7% compounded monthly, paid at the end of the month. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) How much is the monthly payment?

(b) How much of the first month's payment is applied to paying off the principal?

(c) What is the unpaid balance at the end of 25 years?

(d) How much of the principal is repaid during the 26th year?

(e) How much interest is paid during the 301st month?

12. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
A single die is tossed 20 times. Find the probability that exactly 9 of the tosses show either "four" or "five", (Round your answer to 4 decimal places.)
13. (20 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!**

Suppose that the following data were obtained from the records of a certain two-year college. Of those who were freshmen (F) during a particular year, 60% became sophomores (S) the next year and 40% dropped out (D). Of those who were sophomores during a particular year, 75% graduated (G) by the next year and 25% dropped out. (Round your answers to four decimal places.)

(a) Set up the absorbing stochastic matrix with states D, G, F, S that describes this transition.

\[
\begin{bmatrix}
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S}
\end{bmatrix}
\]

(b) Find R, S and Fundamental matrix.

\[R = \begin{bmatrix}
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S}
\end{bmatrix}, \quad S = \begin{bmatrix}
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S}
\end{bmatrix}
\]

\[F = \begin{bmatrix}
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S}
\end{bmatrix}
\]

(c) Find the stable matrix.

\[
\begin{bmatrix}
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S}
\end{bmatrix}
\]

(d) If there are 900 freshmen and 600 sophomores at the beginning of the year, how many of them will drop out at the end of the year?
Statement of Ethics regarding this exam
I agree to complete this exam without unauthorized assistance from any person, materials, or device.
Signature: __________________________ Date: 6 May 2013

DIRECTIONS, Read Carefully

- This exam consists of 13 questions worth 160 points. Answer all questions.

- It is your responsibility to make sure that all 10 pages are present.

- The exam is closed book. No extra papers are allowed. If you need extra paper your instructor will give you additional paper. Return the additional paper with your exam paper and write your name.

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- Turn off your cell phone or any other wireless device or put it into your bag in silent mode.

- Follow the instructions to receive full credit. Just answers or irrelevant calculations receive no credit.

- Remember the ethics policy.

- Best Wishes!

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</table>

VERSION F
Chapter 5

Let A, B be two sets.

Inclusion Exclusion principle

\[ n(A \cup B) = n(A) + n(B) - n(A \cap B). \]

Complement Rule

\[ n(A) = 1 - n(A'). \]

De Morgan’s Laws Let S and T be sets. Then

\[ (S \cap T)' = S' \cup T' \]
\[ (S \cup T)' = S' \cap T' \]
Chapter 6
Let $A, B$ be two events in a sample space.

Inclusion Exclusion principle

$$
\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B).
$$

Complement Rule

$$
\Pr(A) = 1 - \Pr(A').
$$

Conditional Probability If $\Pr(B) \neq 0$. Then

$$
\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}.
$$

Product Rule If $\Pr(B) \neq 0$, then

$$
\Pr(A \cap B) = \Pr(B) \Pr(A \mid B).
$$

Independence $A$ and $B$ are independent if

$$
\Pr(A \cap B) = \Pr(A) \Pr(B).
$$

Chapter 7
Probability Distribution
Let $X$ be a random variable with numerical outcomes $x_1, x_2, \ldots, x_N$ with

$$
\Pr(X = x_1) = p_1, \Pr(X = x_2) = p_2, \ldots, \Pr(X = x_N) = p_N.
$$

Then

$$
\mu = E(X) = x_1 p_1 + x_2 p_2 + \cdots + x_N p_N.
$$

Variance $\sigma^2 = (x_1 - \mu)^2 p_1 + (x_2 - \mu)^2 p_2 + \cdots + (x_N - \mu)^2 p_N$.

Binomial Distribution
$X$ is a binomial random variable with parameters $n$ and $p$, then

Binomial Coefficient

$$
\binom{n}{k} = \frac{n!}{k!(n-k)!}.
$$

Binomial Probability

$$
\Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}.
$$

$$
E(X) = \mu = np, \quad \sigma = \sqrt{np(1-p)}.
$$

$$
\Pr(r \leq X \leq s) = \text{sum(seq(binompdf(n, p, X), X, r, s))}.
$$
Chebychev’s Inequality: Suppose that a probability distribution with numerical outcomes has expected value $\mu$ and standard deviation $\sigma$. Then the probability that a randomly chosen outcome lies between $\mu - c$ and $\mu + c$ is at least

$$1 - \left(\frac{\sigma}{c}\right)^2.$$ 

Chapter 10
Present value $P$, future value $F$, interest compounded at a rate $i$ per period

$$F = (1 + i)^n P$$

Increasing Annuity $n$ payments $R$ each, interest compounded at a rate $i$ per period. Future value $F$

$$F = \frac{(1 + i)^n - 1}{i} R$$

Decreasing Annuity $n$ payments $R$ each, interest compounded at a rate $i$ per period. Present value $P$

$$P = \frac{(1 + i)^n - 1}{i(1 + i)^n} R$$

1. (10 points) Let

$$A = \begin{bmatrix} 9 & -3 \\ -4 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 7 \\ 4 & -2 \end{bmatrix}$$

Find the following.

(a) $A + B = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$

(b) $AB = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$

(c) $B - A = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$
2. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Each of five people randomly chooses one of five calculus sections to take (A, B, C, D, or E). (Round your answers to four decimal places.)
(a) What is the probability that they all choose the same one?
(b) What is the probability that they each choose a different section?

3. (10 points) After applying the Gauss-Jordan elimination, a system equations have been reduced to the following.
(a) Find all solutions (if there is any), of the following system.
\[
\begin{bmatrix}
1 & 3 & 0 & 4 \\
0 & 0 & 1 & 5 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
x = ____________, y = ____________, z = ____________
(b) Find all solutions (if there is any), of the following system.
\[
\begin{bmatrix}
1 & 3 & 0 & 4 \\
0 & 0 & 1 & 5 \\
0 & 0 & 0 & 6
\end{bmatrix}
\]
x = ____________, y = ____________, z = ____________
4. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COM-
MANDS THAT YOU USE!
Consider the following system of equations.
\[
\begin{align*}
2x + 3y + 2z &= 3 \\
3x + 2y - 2z &= 2 \\
x - y - 3z &= 1
\end{align*}
\]
(a) Write the corresponding augmented matrix.
\[
\begin{bmatrix}
& & & \\
& & & \\
& & & \\
& & & \\
& & & \\
& & & \\
& & & \\
\end{bmatrix}
\]
(b) Transform the matrix to the diagonal form (if possible.)
\[
\begin{bmatrix}
& & & \\
& & & \\
& & & \\
& & & \\
& & & \\
& & & \\
& & & \\
\end{bmatrix}
\]
(c) Find all solution.
\[
x = \quad , \\
y = \quad , \\
z = \quad 
\]

5. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COM-
MANDS THAT YOU USE!
A plumbing-supplies manufacturer produces faucet washers that are packaged in boxes of
500. Quality control studies have shown that 8% of the washers are defective. Approximate the probability using normal approximation that more than 40 of the washers in a
single box are defective? Check the rule of thumb. (Round your answer to 4 decimal
places.)
6. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Suppose that the lifetimes of a certain light bulb are normally distributed with \( \mu = 3000 \) hours and \( \sigma = 500 \). Find the probability that a light bulb will burn out in less than 2700 hours. (Round your answers to four decimal places.)

7. (10 points) YOU MUST SHOW YOUR WORK
A universal set \( U \) has three subsets, \( A, B, \) and \( C \). The numbers of elements in various subsets of \( U \) are given below. Find \( n(A \cup B') \)? Show the number of elements in each fundamental regions on the venn diagram.

\[
\begin{align*}
n(U) &= 500 \\
n(A) &= 150 \\
n(B) &= 140 \\
n(C) &= 120 \\
n(A \cap B) &= 80 \\
n(B \cap C) &= 60 \\
n(A \cap C) &= 70 \\
n(A \cap B \cap C) &= 40
\end{align*}
\]

8. (10 points) YOU MUST SHOW ALL FORMULAS THAT YOU USE!
A code is formed by selecting four letters from the set \( L = \{A, B, C, D, E, F, G, H, I, J\} \), without replacement. For example, JEAFG and HGBFD are two possible codes, but CAAFD is not a possible code. How many such codes will contain the letter \( J \)? (Round your answers to four decimal places.)
9. (15 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Twelve thousand dollars is deposited in a savings account at 7% interest compounded monthly. (Round your answer to 2 decimal places, i.e., nearest cents.)
(a) Find the balance after 15 years.

(b) Find the amount of interest earned during that time.

(c) Find the effective rate.

10. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
The table below shows the number of advanced degrees (in thousands) earned in the U.S. during a recent year by gender and type of degree. Find the probability of a person selected at random who received an advanced degree with the following restrictions. (Round your answers to four decimal places.)

<table>
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<tr>
<th></th>
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<th>Doctor’s</th>
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<td>573</td>
<td>211</td>
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<tr>
<td>Female</td>
<td>775</td>
<td>301</td>
<td>22</td>
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</table>

(a) The person is female, given that they received a master’s degree.

(b) The person has received a bachelor’s degree, given that they are male.
11. (25 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Consider a $80,000 mortgage, paid in 30 years at interest rate 5% compounded monthly, paid at the end of the month. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) How much is the monthly payment?

(b) How much of the first month's payment is applied to paying off the principal?

(c) What is the unpaid balance at the end of 25 years?

(d) How much of the principal is repaid during the 26th year?

(e) How much interest is paid during the 301st month?

12. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
A single die is tossed 21 times. Find the probability that exactly 10 of the tosses show either "four" or "five", (Round your answer to 4 decimal places.)
13. (20 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COM-
MANDS THAT YOU USE!
Suppose that the following data were obtained from the records of a certain two-year col-
lege. Of those who were freshmen (F) during a particular year, 65% became sophomores
(S) the next year and 35% dropped out (D). Of those who were sophomores during a par-
ticular year, 70% graduated (G) by the next year and 30% dropped out. (Round your
answers to four decimal places.)

(a) Set up the absorbing stochastic matrix with states D, G, F, S that describes this
transition.

\[
\begin{bmatrix}
\text{D} & \text{G} & \text{F} & \text{S} \\
\end{bmatrix}
\]

(b) Find R, S and Fundamental matrix.
\[R = \begin{bmatrix}
\text{D} & \text{G} \\
\end{bmatrix}, \quad S = \begin{bmatrix}
\text{F} & \text{S} \\
\end{bmatrix}\]
\[F = \begin{bmatrix}
\text{D} & \text{G} \\
\end{bmatrix}\]

(c) Find the stable matrix.
\[
\begin{bmatrix}
\text{D} & \text{G} & \text{F} & \text{S} \\
\end{bmatrix}
\]

(d) If there are 800 freshmen and 500 sophomores at the beginning of the year, how many
of them will drop out at the end of the year?
Statement of Ethics regarding this exam
I agree to complete this exam without unauthorized assistance from any person, materials, or device.
Signature: __________________________ Date: 6 May 2013

DIRECTIONS, Read Carefully

- This exam consists of 13 questions worth 160 points. Answer all questions.
- It is your responsibility to make sure that all 10 pages are present.
- The exam is closed book. No extra papers are allowed. If you need extra paper your instructor will give you additional paper. Return the additional paper with your exam paper and write your name.
- Only TI83-TI84 calculators are allowed.
- Turn off your cell phone or any other wireless device or put it into your bag in silent mode.
- Follow the instructions to receive full credit. Just answers or irrelevant calculations receive no credit.
- Remember the ethics policy.
- Best Wishes!

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<td><strong>Total:</strong></td>
<td><strong>160</strong></td>
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### Chapter 5

Let $A$, $B$ be two sets.

**Inclusion Exclusion principle**

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

**Complement Rule**

$$n(A) = 1 - n(A').$$

**De Morgan’s Laws** Let $S$ and $T$ be sets. Then

$$(S \cap T)' = S' \cup T'$$

$$(S \cup T)' = S' \cap T'.$$
Chapter 6
Let $A, B$ be two events in a sample space.

Inclusion Exclusion principle

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B).$$

Complement Rule

$$\Pr(A) = 1 - \Pr(A').$$

Conditional Probability If $\Pr(B) \neq 0$. Then

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}.$$

Product Rule If $\Pr(B) \neq 0$, then

$$\Pr(A \cap B) = \Pr(B) \Pr(A \mid B).$$

Independence $A$ and $B$ are independent if

$$\Pr(A \cap B) = \Pr(A) \Pr(B).$$

Chapter 7

Probability Distribution
Let $X$ be a random variable with numerical outcomes $x_1, x_2, \ldots, x_N$ with

$$\Pr(X = x_1) = p_1, \Pr(X = x_2) = p_2, \ldots, \Pr(X = x_N) = p_N.$$ Then

$$\mu = E(X) = x_1p_1 + x_2p_2 + \cdots + x_Np_N.$$

Variance

$$\sigma^2 = (x_1 - \mu)^2p_1 + (x_2 - \mu)^2p_2 + \cdots + (x_N - \mu)^2p_N.$$

Binomial Distribution

$X$ is a binomial random variable with parameters $n$ and $p$, then

Binomial Coefficient

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

Binomial Probability

$$\Pr(X = k) = \binom{n}{k}p^k(1-p)^{n-k}.$$

$$E(X) = \mu = np, \quad \sigma = \sqrt{np(1-p)}.$$

$$\Pr(r \leq X \leq s) = \text{sum(seq(binopdf(n,p,X),X,r,s))}.$$
Chebychev’s Inequality: Suppose that a probability distribution with numerical outcomes has expected value $\mu$ and standard deviation $\sigma$. Then the probability that a randomly chosen outcome lies between $\mu - c$ and $\mu + c$ is at least

$$1 - (\frac{\sigma}{c})^2.$$
2. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Each of six people randomly chooses one of six calculus sections to take (A, B, C, D, E, F). (Round your answers to four decimal places.)

(a) What is the probability that they all choose the same one?

(b) What is the probability that they each choose a different section?

3. (10 points) After applying the Gauss-Jordan elimination, a system equations have been reduced to the following.

(a) Find all solutions (if there is any), of the following system.

\[
\begin{bmatrix}
1 & 4 & 0 & 3 \\
0 & 0 & 1 & 6 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

\[x = ____________, \quad y = ____________, \quad z = ____________

(b) Find all solutions (if there is any), of the following system.

\[
\begin{bmatrix}
1 & 4 & 0 & 3 \\
0 & 0 & 1 & 3 \\
0 & 0 & 0 & 4
\end{bmatrix}
\]

\[x = ____________, \quad y = ____________, \quad z = ____________

4. (10 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!**

Consider the following system of equations.

\[
\begin{cases}
5x + 4y + z = 1 \\
4x - 5y + 2z = 8 \\
-x + y - z = 1
\end{cases}
\]

(a) Write the corresponding augmented matrix.

\[
\begin{bmatrix}
\_ & \_ & \_ & | & \\
\_ & \_ & \_ & | \\
\_ & \_ & \_ & |
\end{bmatrix}
\]

(b) Transform the matrix to the diagonal form (if possible.)

\[
\begin{bmatrix}
\_ & \_ & \_ & | \\
\_ & \_ & \_ & |
\end{bmatrix}
\]

(c) Find all solution.

\[x = \_ , \ y = \_ , \ z = \_\]

5. (10 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!**

A plumbing-supplies manufacturer produces faucet washers that are packaged in boxes of 600. Quality control studies have shown that 7% of the washers are defective. Approximate the probability using normal approximation that more than 45 of the washers in a single box are defective? Check the rule of thumb. *(Round your answer to 4 decimal places.)*
6. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Suppose that the lifetimes of a certain light bulb are normally distributed with \( \mu = 2500 \) hours and \( \sigma = 400 \). Find the probability that a light bulb will burn out in less than 2600 hours. (Round your answers to four decimal places.)

7. (10 points) YOU MUST SHOW YOUR WORK
A universal set \( U \) has three subsets, \( A \), \( B \), and \( C \). The numbers of elements in various subsets of \( U \) are given below. Find \( n(A \cup B') \)? Show the number of elements in each fundamental regions on the venn diagram.

\[ n(U) = 500 \quad n(A \cap B) = 70 \]
\[ n(A) = 140 \quad n(B \cap C) = 80 \]
\[ n(B) = 150 \quad n(A \cap C) = 60 \]
\[ n(C) = 120 \quad n(A \cap B \cap C) = 20 \]

8. (10 points) YOU MUST SHOW ALL FORMULAS THAT YOU USE!
A code is formed by selecting six letters from the set \( L = \{A, B, C, D, E, F, G, H, I, J\} \), without replacement. For example, JEAFGH and AHGBFD are two possible codes, but ICAAFD is not a possible code. How many such codes will contain the letter \( J \) ? (Round your answers to four decimal places.)
9. (15 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Thirteen thousand dollars is deposited in a savings account at 8% interest compounded monthly. (Round your answer to 2 decimal places, i.e., nearest cents.)
(a) Find the balance after 16 years.

(b) Find the amount of interest earned during that time.

(c) Find the effective rate.

10. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
The table below shows the number of advanced degrees (in thousands) earned in the U.S. during a recent year by gender and type of degree. Find the probability of a person selected at random who received an advanced degree with the following restrictions. (Round your answers to four decimal places.)

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(a) The person is male, given that they received a master's degree.

(b) The person has received a bachelor's degree, given that they are female.
11. (25 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COM­MANDS THAT YOU USE!
Consider a $90,000 mortgage, paid in 30 year at interest rate 11% compounded monthly, paid at the end of the month. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) How much is the monthly payment?

(b) How much of the first month’s payment is applied to paying off the principal?

(c) What is the unpaid balance at the end of 25 years?

(d) How much of the principal is repaid during the 26th year?

(e) How much interest is paid during the 301st month?

12. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COM­MANDS THAT YOU USE!
A single die is tossed 22 times. Find the probability that exactly 11 of the tosses show either "four" or "five", (Round your answer to 4 decimal places.)
13. (20 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COM-
MANDS THAT YOU USE!
Suppose that the following data were obtained from the records of a certain two-year college. Of those who were freshmen (F) during a particular year, 65% became sophomores (S) the next year and 35% dropped out (D). Of those who were sophomores during a particular year, 80% graduated (G) by the next year and 20% dropped out. (Round your answers to four decimal places.)

(a) Set up the absorbing stochastic matrix with states D, G, F, S that describes this transition.

\[
\begin{bmatrix}
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S}
\end{bmatrix}
\]

(b) Find R, S and Fundamental matrix.
\[
R = \begin{bmatrix} \text{D} & \text{G} \\ \text{D} & \text{G} \end{bmatrix}, \quad S = \begin{bmatrix} \text{D} & \text{G} \\ \text{D} & \text{G} \end{bmatrix}
\]

\[
F = \begin{bmatrix} \text{D} & \text{G} \\ \text{D} & \text{G} \end{bmatrix}
\]

(c) Find the stable matrix.

\[
\begin{bmatrix}
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S} \\
\text{D} & \text{G} & \text{F} & \text{S}
\end{bmatrix}
\]

(d) If there are 700 freshmen and 600 sophomores at the beginning of the year, how many of them will drop out at the end of the year?
Statement of Ethics regarding this exam
I agree to complete this exam without unauthorized assistance from any person, materials, or device.
Signature: __________________________ Date: 6 May 2013

DIRECTIONS, Read Carefully

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VERSION H
Chapter 5
Let $A$, $B$ be two sets.

Inclusion Exclusion principle

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

Complement Rule

$$n(A) = 1 - n(A').$$

De Morgan's Laws Let $S$ and $T$ be sets. Then

$$ (S \cap T)' = S' \cup T'$$
$$ (S \cup T)' = S' \cap T'$$

Table 1: Areas under the standard normal curve

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Chapter 6

Let $A$, $B$ be two events in a sample space.

Inclusion Exclusion principle

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B).$$

Complement Rule

$$\Pr(A) = 1 - \Pr(A').$$

Conditional Probability If $\Pr(B) \neq 0$. Then

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}.$$  

Product Rule If $\Pr(B) \neq 0$, then

$$\Pr(A \cap B) = \Pr(B) \Pr(A \mid B).$$

Independence $A$ and $B$ are independent if

$$\Pr(A \cap B) = \Pr(A) \Pr(B).$$

Chapter 7

Probability Distribution

Let $X$ be a random variable with numerical outcomes $x_1, x_2, \ldots, x_N$ with

$$\Pr(X = x_1) = p_1, \Pr(X = x_2) = p_2, \ldots, \Pr(X = x_N) = p_N.$$  

Then

$$\mu = E(X) = x_1p_1 + x_2p_2 + \cdots + x_Np_N.$$  

Variance $\sigma^2 = (x_1 - \mu)^2p_1 + (x_2 - \mu)^2p_2 + \cdots + (x_N - \mu)^2p_N.$

Binomial Distribution

$X$ is a binomial random variable with parameters $n$ and $p$, then

Binomial Coefficient

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$  

Binomial Probability

$$\Pr(X = k) = \binom{n}{k}p^k(1-p)^{n-k}.$$  

$$E(X) = \mu = np, \quad \sigma = \sqrt{np(1-p)}.$$  

$$\Pr(r \leq X \leq s) = \text{sum(seq(binompdf(n,p,X),X,r,s)).}$$
Chebyshev’s Inequality: Suppose that a probability distribution with numerical outcomes has expected value $\mu$ and standard deviation $\sigma$. Then the probability that a randomly chosen outcome lies between $\mu - c$ and $\mu + c$ is at least

$$1 - \left(\frac{\sigma}{c}\right)^2.$$
2. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Each of seven people randomly chooses one of seven calculus sections to take (A, B, C, D, E, F, G). (Round your answers to four decimal places.)
(a) What is the probability that they all choose the same one?

(b) What is the probability that they each choose a different section?

3. (10 points) After applying the Gauss-Jordan elimination, a system equations have been reduced to the following.
(a) Find all solutions (if there is any), of the following system.
\[
\begin{bmatrix}
1 & 2 & 0 & 9 \\
0 & 0 & 1 & 4 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
\[x = \phantom{000}, \quad y = \phantom{00}, \quad z = \phantom{00}\]

(b) Find all solutions (if there is any), of the following system.
\[
\begin{bmatrix}
1 & 2 & 0 & 9 \\
0 & 0 & 1 & 4 \\
0 & 0 & 0 & 11
\end{bmatrix}
\]
\[x = \phantom{000}, \quad y = \phantom{00}, \quad z = \phantom{00}\]
4. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
Consider the following system of equations.
\[
\begin{align*}
2x - 3y + z &= 4 \\
3x - 2y + z &= 9 \\
x + y - z &= 1
\end{align*}
\]
(a) Write the corresponding augmented matrix.
\[
\begin{bmatrix}
2 & -3 & 1 & | & 4 \\
3 & -2 & 1 & | & 9 \\
1 & 1 & -1 & | & 1
\end{bmatrix}
\]
(b) Transform the matrix to the diagonal form (if possible.)
\[
\begin{bmatrix}
1 & 0 & 0 & | & 1 \\
0 & 1 & 0 & | & 2 \\
0 & 0 & 1 & | & 3
\end{bmatrix}
\]
(c) Find all solution.
\[
x = \_\_\_\_, \quad y = \_\_\_, \quad z = \_\_\_
\]

5. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!
A plumbing-supplies manufacturer produces faucet washers that are packaged in boxes of 700. Quality control studies have shown that 5% of the washers are defective. Approximate the probability using normal approximation that more than 40 of the washers in a single box are defective? Check the rule of thumb. (Round your answer to 4 decimal places.)
6. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COM-
MANDS THAT YOU USE!
Suppose that the lifetimes of a certain light bulb are normally distributed with $\mu = 4000$
hours and $\sigma = 300$. Find the probability that a light bulb will burn out in less than 4200
hours. (Round your answers to four decimal places.)

7. (10 points) YOU MUST SHOW YOUR WORK
A universal set $U$ has three subsets, $A$, $B$, and $C$. The numbers of elements in various
subsets of $U$ are given below. Find $n(A \cup B')$? Show the number of elements in each
fundamental regions on the venn diagram.

$$
\begin{align*}
n(U) &= 500 \\
n(A) &= 180 \\
n(B) &= 120 \\
n(C) &= 100 \\
n(A \cap B) &= 90 \\
n(B \cap C) &= 80 \\
n(A \cap C) &= 50 \\
n(A \cap B \cap C) &= 30
\end{align*}
$$

8. (10 points) YOU MUST SHOW ALL FORMULAS THAT YOU USE!
A code is formed by selecting seven letters from the set $L = \{A, B, C, D, E, F, G, H, I, J\}$,
without replacement. For example, JEAFGHB and ACHGBFD are two possible codes,
but ICAAFFDB is not a possible code. How many such codes will contain the letter $J$
?(Round your answers to four decimal places.)
9. (15 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COM­MANDS THAT YOU USE!
Fourteen thousand dollars is deposited in a savings account at 9% interest compounded monthly. (Round your answer to 2 decimal places, i.e., nearest cents.)
(a) Find the balance after 17 years.

(b) Find the amount of interest earned during that time.

(c) Find the effective rate.

10. (10 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COM­MANDS THAT YOU USE!
The table below shows the number of advanced degrees (in thousands) earned in the U.S. during a recent year by gender and type of degree. Find the probability of a person selected at random who received an advanced degree with the following restrictions. (Round your answers to four decimal places.)

<table>
<thead>
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<th>Gender</th>
<th>Bachelor's</th>
<th>Master's</th>
<th>Doctor's</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>573</td>
<td>211</td>
<td>24</td>
</tr>
<tr>
<td>Female</td>
<td>775</td>
<td>301</td>
<td>22</td>
</tr>
</tbody>
</table>

(a) The person is male, given that they received a bachelor's degree.

(b) The person has received a master's degree, given that they are female.
11. (25 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!**
Consider a $100,000 mortgage, paid in 30 year at interest rate 13% compounded monthly, paid at the end of the month. (Round your answer to 2 decimal places, i.e., nearest cents.)

(a) How much is the monthly payment?

(b) How much of the first month's payment is applied to paying off the principal?

(c) What is the unpaid balance at the end of 25 years?

(d) How much of the principal is repaid during the 26th year?

(e) How much interest is paid during the 301st month?

12. (10 points) **YOU MUST SHOW ALL FORMULAS or CALCULATOR COMMANDS THAT YOU USE!**
A single die is tossed 23 times. Find the probability that exactly 12 of the tosses show either "four" or "five", (Round your answer to 4 decimal places.)
13. (20 points) YOU MUST SHOW ALL FORMULAS or CALCULATOR COM-
MANDS THAT YOU USE!
Suppose that the following data were obtained from the records of a certain two-year col-
lege. Of those who were freshmen (F) during a particular year, 60% became sophomores (S) the next year and 40% dropped out (D). Of those who were sophomores during a par-
ticular year, 85% graduated (G) by the next year and 15% dropped out. (Round your
answers to four decimal places.)

(a) Set up the absorbing stochastic matrix with states D, G, F, S that describes this
transition.

\[
\begin{bmatrix}
\_ & \_ & \_ & \_ \\
\_ & \_ & \_ & \_ \\
\_ & \_ & \_ & \_ \\
\_ & \_ & \_ & \_ \\
\end{bmatrix}
\]

(b) Find R, S and Fundamental matrix.

\[
R = \begin{bmatrix}
\_ & \_ \\
\_ & \_ \\
\_ & \_ \\
\end{bmatrix}, \quad S = \begin{bmatrix}
\_ & \_ \\
\_ & \_ \\
\_ & \_ \\
\_ & \_ \\
\end{bmatrix}
\]

\[
F = \begin{bmatrix}
\_ & \_ \\
\_ & \_ \\
\_ & \_ \\
\end{bmatrix}
\]

(c) Find the stable matrix.

\[
\begin{bmatrix}
\_ & \_ & \_ & \_ \\
\_ & \_ & \_ & \_ \\
\_ & \_ & \_ & \_ \\
\end{bmatrix}
\]

(d) If there are 600 freshmen and 700 sophomores at the beginning of the year, how many
of them will drop out at the end of the year?