**MAT 117 Final Exam Fall 2014**  
**December 10, 2014**

**Directions:** Answer all of the following questions. Some questions may have multiple parts. Be sure to show all of your work. Unsupported claims will receive little or no credit. You may use a calculator (not the one on your cell phone). There should be no collaboration with anyone else. Be sure to explain your answers as clearly as you can. If you need more space to work on a problem, clearly indicate where the solution to the problem can be found.

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Total Score: _______/130
1. (6 points) In the following sequence, find the next three terms. Explain the pattern.

4, 7, 12, 19, 28, _____, _____, _____

2. (10 points)
   a. Find the following difference using the Austrian Subtraction Algorithm (also known as the equal addition algorithm). Be sure to show all of your work and the steps involved in your process!

   \[
   \begin{array}{cccc}
   7 & 3 & 2 \\
   -1 & 7 & 3 \\
   \end{array}
   \]

   b. Why does this method of subtraction work?

   c. Find the following sum using the Scratch Addition Algorithm. Be sure to show all of your work and the steps involved in your process!

   \[
   \begin{array}{cccc}
   8 & 4 & 1 \\
   2 & 5 & 6 \\
   3 & 6 & 6 \\
   +1 & 7 & 3 \\
   \end{array}
   \]

   d. Why does this method of addition work?
3. (12 points) True or False? If true, explain why. If false, explain why OR give a counterexample.

a. The set of integers is closed under subtraction.

b. If a number is divisible by 3 and by 6, then it is also divisible by 18.

c. The probability of rolling a 2 or a 5 on a six-sided die is equal to the product of the probabilities of rolling each number: \( \frac{1}{6} \cdot \frac{1}{6} \).

d. A two digit number in base 6 is always greater than a three digit number in base 3.

e. The mean of a data set can be found on a box-and-whisker plot.
4. (9 points)

   a. Convert the base-ten numeral 197 to base three.

   b. Convert the base-two numeral 111010001\textsubscript{two} to base ten.

   c. Convert the base-four numeral 31\textsubscript{four} to base two.
5. (6 points) Find the missing digits in the following:

a. \[ \begin{array}{cccc}
1 & \_ & 1 & \_ \\
+ & 1 & 1 & \_ \\
\hline
& 1 & 0 & \_ \\
\end{array} \]

b. \[ \begin{array}{cccc}
4 & 5 & 2 & \_ \\
- & 1 & \_ & 3 \\
\hline
& 0 & \_ & \_ \\
\end{array} \]

7. (12 points)
   a. How many factors does a number of the form \( p^2 \cdot q \cdot r \), where \( p, q, \) and \( r \) are prime, have?

   b. In how many ways can a sorority of 60 members select a president, vice president and treasurer, assuming that the same person cannot hold more than one office.

   c. There are 20 female students and three male students in one section of MAT 117. How many groups of four females can be chosen?

   d. Anna wants to set a password so that her younger sister cannot get into her computer. The password consists of 3 letters, followed by a 3 digit number. If Anna wants the first number of the password to be 7 (her favorite number), how many options does she have for the password?
8. (12 points) A bag contains 3 red balls, 2 blue balls, and 1 white ball. Two balls are drawn, one after the other, without replacement and the color of each ball is recorded.

   a. Draw a tree diagram representing this experiment. Be sure to put the probabilities on the branches.

   b. List the sample space of the experiment.

   c. What is the probability of drawing two balls of the same color?

   d. What is the probability that the first ball is red OR blue?
9. (9 points)
a. Is the set of odd numbers \( \{1, 3, 5, 7, 9\ldots \} \) closed under addition? Is it closed under multiplication? Explain.

b. Does the set of natural numbers have an additive identity? Explain why or why not.

c. Use an example to show why multiplicative inverses are not a part of the set of integers.

10. (9 points) Perform the following operations modulo the number indicated in the parentheses. Explain how you got your answer.

a. \( 28 + 3 \pmod{4} \)

b. \( 2 - 4 \pmod{5} \)

c. \( 2 + 5 \pmod{6} \)
11. (9 points) The selling prices of 10 homes in a particular neighborhood are given by:

120,000 135,000 105,000 400,000 112,000
125,000 107,000 125,000 125,000 115,000

a. What measure would you use to describe the “typical” selling price of a home in that neighborhood? Why?

b. What value does that measure give as the “typical” selling price?

c. Sketch a box-and-whisker plot for these data. Be sure to include a scale on your axis.