Qualifying Exam, Complex Analysis, January 12, 2018

Notation: Throughout the exam $\Delta$ denotes the open unit disc in $\mathbb{C}$.

1. Find a conformal map from the strip $\{0 < \text{Im} z < \pi \}$ onto $\Delta$.

2. Find $\int_{|z|=7} \frac{\sin z}{4z^2 - \pi^2} \, dz$.

3. Let $f$ be a non-constant entire function. Show that the function $e^f$ has an isolated essential singularity at infinity.

4. If $f(z) = \frac{1 + z^2}{1 - z^2}$, find $f(\Delta)$.