1. For each \( n \geq 1 \), let \( X_n \) have density
\[
f_n(x) = \frac{n}{\pi(1 + n^2x^2)}, \quad -\infty < x < \infty.
\]
(a) Does \( X_n \to 0 \) in probability?
(b) If \( X_n, n \geq 1 \) are defined on the same probability space and are independent, does \( X_n \to 0 \) a.s.?

2. Let \( X_1, X_2, \ldots \) be non-degenerate IID random variables such that \( P(X_1 > 0) = 1 \) and \( EX_1 = 1 \). Prove that
\[
\lim_{n \to \infty} \prod_{i=1}^{n} X_i = 0 \text{ a.s}
\]

3. Let \( X_k, k \geq 1 \) be an IID sequence of non-degenerate random variables with \( X_1 \in L^\infty \). For each \( \alpha \geq -1/2 \) show that there exist norming constants \( a_n \) and \( b_n \) such that
\[
S_n = \frac{(\sum_{k=1}^{n} k^{\alpha} X_k) - a_n}{b_n}
\]
converges in distribution to a standard normal distribution.

4. Let \( X_1, X_2 \) be non-degenerate IID random variables with mean 0 and finite second moment. Assume \( \frac{X_1 + X_2}{c} \) and \( X_1 \) have the same distribution.
(a) Find the value of \( c \).
(b) Prove that \( X_1 \) has a normal distribution.