MAT 285   Final Exam 2013   Ver. A

SU ID#:  

Signature:  

Instructions: Write the answers and show the main steps of your work on this test sheet. 

There are 13 questions on 12 pages. Be sure you have all 12 pages (6 sheets) 

You should spend no more than 35 minutes on each Part; be sure that you get to the easier parts of each problem. 

All cell phones / head phones must be turned off and put away. 

The Final Exam is scored on a basis of 100 points and will count 20% of your final grade. 

You must show your work to get full credit! 

DO NOT WRITE ON THE REST OF THIS COVER SHEET!  

Problem 1 (5)    Problem 6 (12)    Problem 9 (6)  
Problem 2 (5)    Problem 7 (9)     Problem 10 (3) 
Problem 3 (10)   Problem 8 (12)    Problem 11 (12) 
Problem 4 (7)    Problem 12 (7)    Problem 13 (5)  
Test 1 Total (34)       Test 2 Total (33)       Test 3 Total (33)  

EXAM TOTAL (100)
PART 1

Problem 1. (5 points) Find the equation of the line through the point \((-4, 3)\) with slope \(\frac{1}{2}\).

Problem 2. (5 points) Describe the domain of \(f(x) = \sqrt{25-x^2} \div (x-3)\).

Is \(f(x)\) continuous at \(x = 0\) CIRCLE ONE: YES NO
Problem 3. (10 points)

(i) Solve \( \log_2(x + 3) = 2 \) for \( x \)

(ii) Solve \( \sin\left(\frac{\pi}{2} + x\right) = 0 \) for \(-\pi \leq x \leq \pi\)
Problem 4. (7 points) Consider the equation \( f(t) = 800e^{kt} \) where 800 is the initial value, \( k \) is a constant and the variable \( t \) is time measured in months.

(i) For what values of \( k \) will this be a growth function?

CIRCLE ONE: \( k < 0 \quad k > 0 \)

(ii) Let \( k = -0.2 \) and find the number of months needed for this function to decrease to half of its initial value.
Problem 5. (7 points) Using the definition of the derivative as a limit compute the derivative of $f(x) = x^2 - 7x + 5$. You MUST show EVERY STEP OF YOUR WORK TO GET CREDIT ON THIS PROBLEM!
Problem 6. (12 points) Differentiate each of the following functions.

(i) \( x^5 - x + x^{-5} \)

(ii) \( (x^5 - x^2 + 2)(x - 5) \)

(iii) \( \frac{(x^5 - x^2 + 2)}{(x-5)} \)

(iv) \( (x^5 - x^2 + 2)^4 \)
Problem 7. (9 points) Differentiate each of the following functions.

(i) \( \ln(2 + \sin x) \)

(ii) \( (\cos x)e^{(x^2+1)} \)

(iii) \( \frac{\ln(x)}{e^x} \)
Problem 8. (12 points) The first and second derivatives of $f(x)$ are: $f'(x) = (x - 2)(x + 4)(x - 7)$ and $f''(x) = 3(x - 4.85)(x + 1.51)$

(i) For what values of $x$ is $f$ increasing?

(ii) For what values of $x$ is $f$ decreasing?

(iii) For what values of $x$ is $f$ concave upward?

(iv) For what values of $x$ is $f$ concave downward?

(v) For what values of $x$ does $f$ have relative maxima?

(vi) For what values of $x$ does $f$ have relative minima?
PART 3

Problem 9. (6 points) Given non negative numbers $x$ and $y$ such that $x + y = 3$, maximize $f(x, y) = yx^2$

Problem 10. (3 points) Consider the function $f(x, y) = x^2 + y$

The level curve given by $f(x, y) = -1$ is _____ [FILL IN A, B, C, OR D]

\[ \text{A} \quad \text{B} \quad \text{C} \quad \text{D} \]
Problem 11. (12 points)
The point \( p = (2, -1) \) lies on the curve \( 3x^2 - 2y^3 = 10 \)

(i) Find \( \frac{dy}{dx} \)

(ii) What is the slope of the tangent line to this curve at the point \( p \)

Now think of \( p \) as moving along this curve.

(iii) Compute \( \frac{dy}{dt} \) and \( \frac{dx}{dt} \)

(iv) At the point \( (2, -1) \), \( x \) is changing at the rate of 2ups (unit per second). At what rate is \( y \) changing?
Problem 12. (7 points) Consider the function $f(x, y) = x^2 + y^3 - 12xy$

(i) Compute $f(1, 2)$

(ii) Compute $f_x = \frac{\partial f}{\partial x}$

(iii) Compute $f_y = \frac{\partial f}{\partial y}$

(iv) Compute $f_{xx} = \frac{\partial^2 f}{\partial x^2}$

(v) Compute $f_{yy} = \frac{\partial^2 f}{\partial y^2}$

(vi) Compute $f_{xy} = \frac{\partial^2 f}{\partial x \partial y}$

(vii) Compute $D$
Problem 13. (5 points) If \( f(x, y) = 2x^3 - 6x^2 + y^2 \), we have

\[
\frac{\partial f}{\partial x} = 6x^2 - 12x, \\
\frac{\partial f}{\partial y} = 2y, \\
\frac{\partial^2 f}{\partial x^2} = 12x - 12, \\
\frac{\partial^2 f}{\partial y^2} = 2, \\
\frac{\partial^2 f}{\partial x \partial y} = 0,
\]

and \( D = 24x - 24 \).

(i) Find the coordinates of all points where has a horizontal tangent plane.

(ii) For each of these points decide if \( f \) has a relative maximum, or a relative minimum, or a saddle point or if it is impossible to tell. Explain your answers and show your work.