INSTRUCTIONS

• There are a total of 10 problems. It is your responsibility to make sure that all 10 problems are present.
• Show all your work. Little or no credit will be given for “answers” without supporting work using calculus methods.
• No calculators or other electronic devices are allowed.

Do not write below this line

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1. (12 points) Evaluate each of the following. Write DNE if the limit does not exist. “Answers” without supporting work will receive no credit.

(a) \( \lim_{x \to +\infty} \frac{2x^3 + x + 7}{1 - 3x + 5x^2 + 7x^3} \)

(b) \( \lim_{x \to 0} \frac{\tan 3x}{x^2 + x} \)

(c) \( \lim_{x \to +\infty} x^2 e^{-3x} \)
2. (16 points) Find the derivative of each of the following functions. **DO NOT SIMPLIFY!**

(a) \( f(x) = x^2 (1 + x)^{3/2} \).

(b) \( f(x) = \frac{\cos x}{\cos x + \sin x} \).
(Problem 2 continued)

(c) $f(x) = \tan^{-1}\left(\frac{1}{x}\right)$

(d) $f(x) = (\ln(1 + e^x))^4$
3. (10 points) Answer the following questions for the curve with equation

\[ e^{xy} = x^2 + y^2. \]

(a) Find \( \frac{dy}{dx} \) at the point \((0, 1)\). Simplify your answer.

(b) Find an equation of the tangent line at the point \((0, -1)\).
4. (10 points) The graph of the derivative $f'$ of a function $f$ on the interval $[-2.5, 4.5]$ is shown below.

(a) Find the critical numbers.

(b) On what intervals, if any, is $f$ increasing? Justify your answer.

(c) At what values of $x$, if any, does $f$ have a local minimum? Justify your answer.
5. (6 points) Find the inflection points and intervals of concavity for the function $f(x) = xe^{-3x}$. 


6. (6 points) Find the absolute maximum and minimum of the function \( g(t) = 3t^2 - 2t + 1 \) on the interval \([-2, 2]\).
7. (10 points) An ant moves along the $x$-axis from left to right at 5 inches per second. A spider moves along the $y$-axis from up to down at 3 inches per second. At a certain instant, the ant is 4 inches to the right of the origin and the spider is 8 inches above the origin. At this instant, what is the rate of change of the distance between the spider and the ant?
8. (10 points) A box with a square base and an open top must have a volume of 12 cubic feet.

(a) What dimensions should it have to minimize the amount of material used?

(b) Justify your answer.
9. (16 points) Evaluate the following definite and indefinite integrals.

(a) \[ \int_{1}^{10} \sqrt{z - 1} \, dz \] (Simplify your answer.)

(b) \[ \int \frac{\sin x}{\cos^4 x} \, dx \]

(c) \[ \int (1 + x) \sqrt{x} \, dx \]
(Problem 9 continued)

(d) \( \int_{0}^{2} xe^{x^2} \, dx \)

10. (4 points) Let \( h(x) \) be the function defined by \( h(x) = \int_{1}^{x} t \sin^2(t) \, dt \). Find \( h'(\pi/4) \). Simplify your answer.