### MAT 117 Final Exam Fall 2012
December 12, 2012

Name: ___________________________  Section: ________

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Total Score: ________/130
1. (3 points) In the following sequence, find the next three terms. Explain the pattern.

   2, 5, 10, 17, 26, ___, ___, ___

2. (10 points) True or False? If true, explain why. If false, explain why OR give a counterexample.

   a. The set of integers is closed under subtraction.

   b. All integers can be expressed as a product of prime numbers.

   c. A number is divisible by 8 if the number formed by the two right-most digits is divisible by 8.

   d. The probability of rolling a 2 or a 5 on a six-sided die is equal to the product the probabilities of rolling each number: $1/6 \cdot 1/6$

   e. A two digit number in base 6 is always greater than a three digit number in base three.
3. (8 points) William is laying patio tiles in his backyard. He wants to make a design like the one below and he has 85 tiles to make his design.

![Diagram of tile design]

a. How many tiles need to be placed in the middle row so that he can use the most number of the tiles he purchased? Explain how you determined this.

b. How many total tiles will be in the final structure and how many will be left over? Explain how you determined this.

4. (8 points) Mackenzie was asked to write \((5 \cdot 6^3) + (2 \cdot 6^2) + (4 \cdot 6^1)\) as a base six numeral. She wrote \(524_{six}\).

a. Was she correct? If yes, why? If not, why not? Explain your reasoning and her possible reasoning.
b. Consider the numeration system in use with some machines that uses the combination of our digits and the first six letters of our alphabet. It is assumed that the letters represent ten, eleven, and so on in order. The digits in this system are thus:
{1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F}

i. What is the base of this numeration system? Why?

ii. What number (in base ten) does the numeral AF represent? Why?

5. (5 points) Find the missing digits in the following:

a. \[
\begin{array}{c}
 1 \_ \_ 2 \text{ four} \\
+ 1 1 \_ \_ \text{four} \\
\_ \_ 1 0 \text{ four}
\end{array}
\]

b. \[
\begin{array}{c}
 4 5 \_ 2 \text{ seven} \\
- 1 \_ \_ 3 \text{ seven} \\
\_ 0 \_ \_ \text{ seven}
\end{array}
\]
6. *(12 points)* Complete the multiplication table for mod 5.

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b. Is multiplication commutative in mod 5? Explain.

c. Is there an identity element for multiplication in mod 5? If yes, name it. If no, explain.

d. Does 2 have a multiplicative inverse in mod 5? If yes, name it. If no, explain.

7. *(6 points)* The following is a lattice multiplication showing, where A, B, and C are each a different digit, 0-9 (This is in base 10). Find the values of A, B, and C, and explain how you got your results. Then compute the final product. (Hint: Find the value for A first and then use that value to find B and C.)

```
   0 5 0 1 2 3 4 5
5 |
C |
0 3 4 0 4 0 6
```

```
A
B
C
```
8. *(11 points)* A bag contains 6 red balls, 3 blue balls, and 2 white balls. Two balls are drawn, one after the other, without replacement and the color of each ball is recorded.

a. Draw a tree diagram representing this experiment. Be sure to put the probabilities on the branches.

b. List the sample space of the experiment.

c. What is the probability of drawing two balls of the same color?

d. What is the probability that the first ball is red OR blue?
9. (8 points) Find the following sum using the Austrian Subtraction Algorithm. Be sure to show all your work and the steps involved in your process!

a. 

\[
\begin{array}{c}
\phantom{-}4 & 2 & 0 \\
- & 2 & 8 & 8 \\
\end{array}
\]

b. Why does this method of subtraction work?

10. (6 points) Perform the following operations modulo the number indicated in the parentheses. Explain how you got your answer.

a. \(28 + 3 \pmod{4}\)

b. \(2 - 4 \pmod{5}\)

c. \(2 \div 5 \pmod{6}\)
11. a) (4 points) Is the set of odd numbers {1, 3, 5, 7, 9...} closed under addition? Is it closed under multiplication? Explain.

b) (4 points) Does the set of natural numbers have an additive identity? Explain why or why not.

c) (4 points) Use an example to show why multiplicative inverses are not a part of the set of integers.

12. (8 points) The selling prices of 10 homes in a particular neighborhood are given by:

\[
\begin{align*}
120,000 & \quad 135,000 & \quad 105,000 & \quad 1,000,000 & \quad 112,000 \\
125,000 & \quad 107,000 & \quad 125,000 & \quad 125,000 & \quad 115,000
\end{align*}
\]

a. What measure would you use to describe the “typical” selling price of a home in that neighborhood? Why?

b. What value does that measure give as the “typical” selling price?

c. Sketch a box-and-whisker plot for these data.