## MAT 117 Final Exam Fall 2011
December 14, 2011

Name _______________________________    Section ________

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Total Score: ________/130

Percentage: ________
1. (5 points) In the following sequence, find the next three terms. **Explain the pattern.**

   5, 9, 16, 26, 39, __, __, __

2. (10 points) True or False? If true, explain why. If false, explain why OR give a counterexample.

   a) All whole numbers are integers.

   b) One is a prime number.

   c) If a number is divisible by 3 and by 6, then it is also divisible by 18.

   d) Experimental probability is always less than theoretical probability.

   e) The mean of a data set can be found on a box-and-whisker plot.
3. (8 points) Susan is building a structure out of blocks for her children to climb on. She is using blocks to make the structure in the shape shown below. Right now her structure is 4 blocks high, but she would like to make the structure taller, so that it is 12 blocks high.

a) How many blocks will be in the bottom row of the final structure (12 blocks high)? Explain how you determined this.

b) How many total blocks will be in the final structure? Explain how you determined this.

4. (8 points) Mackenzie was asked to write \(2 \times 8^6 + 7 \times 8^4 + 1 \times 8^3\) as a base eight numeral. She wrote 271.

a) Was she correct? If yes, why? If not, why not? Explain your reasoning and her possible reasoning.
b) Consider the numeration system in use with some machines that uses the combination of our digits and the first six letters of our alphabet. It is assumed that the letters represent ten, eleven, and so on in order. The digits in this system are thus \{0, 1, 2, 3, ..., 9, a, b, ..., f\}.

i) What is the base of this numeration system? Why?

ii) What number (in base ten) does the numeral \(bc\) represent? Why?

5. (15 points) a) Find the missing digits in the following:

i) \[
\begin{array}{c}
& 1 & \quad \underline{2_{\text{four}}} \\
+ & 1 & \quad \underline{1_{\text{four}}} \\
\hline
& 1 & \quad \underline{0_{\text{four}}} \\
\end{array}
\]

ii) \[
\begin{array}{c}
4 & 5 & \quad \underline{2_{\text{seven}}} \\
- & 1 & \quad \underline{3_{\text{seven}}} \\
\hline
& 0 & \quad \underline{0_{\text{seven}}} \\
\end{array}
\]
b) Complete the multiplication table for mod 5.

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i) Is multiplication closed in mod 5? Explain.

ii) Is multiplication commutative in mod 5? Explain.

iii) Is there an identity element for multiplication in mod 5? If yes, name it. If no, explain.

iv) Does 2 have a multiplicative inverse in mod 5? If yes, name it. If no, explain.

6. A bag contains 7 red balls, 3 blue balls, and 2 white balls. Two balls are drawn, one after the other, without replacement and the color of each ball is recorded.

a) (3 points) Draw a tree diagram representing this experiment. Be sure to put the probabilities on the branches.
b) *(2 points)* List the sample space of the experiment.

c) *(2 points)* What is the probability of drawing a blue ball AND a white ball?

d) *(2 points)* What is the probability that the first ball is red OR blue?

7. *(5 points)* The following is a lattice multiplication showing $5BC \times A6$, where A, B, and C are each a different digit, 0-9. (This is in base 10.)

Find the values of A, B, and C, and explain how you got your results. Then compute the final product. (Hint: Find the value for A first and then use that value to find B and C.)
8. (5 points) Suppose that \( a \) is a natural number greater than 2 for which the following congruencies hold:

\[
\begin{align*}
    a & \equiv 1 \pmod{3} \\
    a & \equiv 1 \pmod{8} \\
    a & \equiv 1 \pmod{5}
\end{align*}
\]

Find one possible solution for \( a \).

9. (8 points) Kelly is a third grade teacher. One of her students recently moved to the United States from Austria. On a homework assignment, Kelly is surprised to see that her new student has subtracted 368 from 425 using the method below.

\[
\begin{array}{c}
    4 \quad 2 \quad 5 \\
- \quad 3 \quad 6 \quad 8 \\
\hline
    4 \quad 2 \quad 15 \\
- \quad 3 \quad 7 \quad 8 \\
\hline
    4 \quad 12 \quad 15 \\
- \quad 4 \quad 7 \quad 8 \\
\hline
    5 \quad 5
\end{array}
\]

a) How would this student solve 316 – 278?

b) Why does this student’s method of subtraction work?
10. (6 points) Perform the following operations modulo the number indicated in the parentheses. Explain how you got your answer.

a) $7 + 5 \pmod{3}$

b) $2 - 9 \pmod{8}$

c) $3 \div 5 \pmod{7}$

11. a) (5 points) Given the set $E = \{1, 2, 4, 6, 8, \ldots\}$, explain why this set is closed under multiplication, but not under addition.

b) (4 points) In the set of whole numbers, does 0 have a multiplicative inverse? If yes, state what it is and why it is the inverse. If no, explain why not.
c) *(4 points)* Use an example to explain why the associative property does not hold for subtraction in the set of integers.

12 *(8 points)* The selling prices of 10 homes in a particular neighborhood are given by:

\[
120,000 \quad 135,000 \quad 105,000 \quad 1,000,000 \quad 112,000 \\
125,000 \quad 107,000 \quad 125,000 \quad 125,000 \quad 115,000
\]

a) What measure would you use to describe the "typical" selling price of a home in that neighborhood? Why?

b) What value does that measure give as the "typical" selling price?

c) Sketch box-and-whisker plot for this data