1. Find the image of the half-disc $D = \{ z \in \mathbb{C} : |z| < 1, \text{Im} z > 0 \}$ by the Möbius map $f(z) = \frac{1 + z}{1 - z}$.

2. Let $f$ be a holomorphic function on $\Delta \setminus \{0\}$ such that $|f(z)| > 1$ for all $z \in \Delta \setminus \{0\}$. Show that 0 is an isolated singularity of $f$ which is either removable or a pole.

3. Let $D \subseteq \mathbb{C}$ be a simply connected domain, $z_0 \in D$, and $f : D \rightarrow \Delta$ be a conformal map such that $f(z_0) = 0$. If $g : D \rightarrow \Delta$ is a holomorphic map such that $g(z_0) = 0$, show that $|g'(z_0)| \leq |f'(z_0)|$, and the equality holds if and only if $g$ is a conformal map.

4. Compute $F(w) = \int_{-\infty}^{+\infty} \frac{e^{ix}}{(x-w)^2} \, dx$, where $w \in \mathbb{C} \setminus \mathbb{R}$. (Hint: consider the cases $\text{Im} w > 0$, and $\text{Im} w < 0$, separately.)