Final Exam

NAME:

SU ID:
1. (8 points) Find the general solution for \[ y' - y = t \]

2. (8 points) Solve the initial value problem \[ y'' + 4y = 0, \; y(0) = 1, \; y'(0) = -1 \]
3. (10 points) A tank initially contains 200 gallons of fresh water, but then a salt solution of unknown concentration is poured into the tank at 2 gal/min. The well-stirred mixture flows out of the tank at the same rate. After 120 min, the concentration of salt in the tank is 1.4 lb/gal. What is the concentration of the corresponding brine?
4. (8 points) Calculate $A^T B$, where 

$A = \begin{bmatrix} 1 + i & 2i \\ 2 & 2 - 3i \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 2i & 1 + i \end{bmatrix}$

5. (10 points) Determine whether the matrix is diagonalizable. If it is, determine the matrix $P$ and $D$, if not, explain why.

$A = \begin{bmatrix} 3 & 1 \\ -1 & 5 \end{bmatrix}$
6. (10 points) Use Gauss-Jordan reduction to transform the augmented matrix of the following system to RREF. Then discuss the solutions of the system (i.e., no solutions, a unique solution, or infinitely many solutions).

\begin{align*}
x + 2y + z &= 2 \\
x - y &= 4 \\
2x - y + 2z &= 0 \\
3y + z &= -2
\end{align*}
7. (12 points) Determine whether or not the given set constitutes a vector space. If yes, check it with definition, otherwise, find a counter example.

a). The set of vectors in the first quadrant of the plane

b). The set of pairs of real numbers \((x, y)\) such that \(x \geq y\)

c). The set of 2 \(\times\) 2 matrices with determinant equal to zero.
8. (12 points) Obtain the general solution of the DE by using the method of undetermined coefficients.

\[ y'' - y' - 2y = 5e^{2t} \]
9. (10 points) Find the general solution for the following DE system.

\[
\vec{x}' = \begin{bmatrix} 3 & 2 \\ -8 & -5 \end{bmatrix} \vec{x}
\]
10. (bonus question 5 points) Solve the nonhomogeneous system by decoupling.

\[
\mathbf{x}' = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix} \mathbf{x}' + \begin{bmatrix} 0 \\ e^{-t} \end{bmatrix}
\]
11. (12 points) Solve the IVP using Laplace transform.

\[ y'' - 3y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = 0 \]