NAME:_________________________

<table>
<thead>
<tr>
<th>Question</th>
<th>Grade</th>
<th>(out of)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>15</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>15</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>
The exam will be graded on a partial credit basis. **Answers without supporting work shown on the paper will receive NO credit.**

1. *(15 points)* Let

\[ A = \begin{bmatrix} 2 & -1 & 1 & 3 \\ 8 & -1 & 1 & 7 \\ 1 & 1 & -1 & -1 \end{bmatrix} \]

(a) Find the reduced echelon form of \( A \) (show all your work).

(b) Find a basis for \( \text{Col}A \).

(c) Find a basis for \( \text{Nul}A \).
2. (7 points) Compute a basis for a subspace \( V = \{ (x, y, z, w) \mid -5x - 2y + 4z = w \} \) of \( \mathbb{R}^4 \).

3. (8 points) Consider the system

\[
\begin{bmatrix}
2 & 1 & -2 \\
0 & 1 & -1 \\
3 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\ x_2 \\ x_3
\end{bmatrix}
= 
\begin{bmatrix}
0 \\ 1 \\ 0
\end{bmatrix}.
\]

Solve this system using Cramer’s rule.
4. (10 points) Solve the system

\[
\begin{bmatrix}
0 & 1 & 2 \\
-2 & 0 & 2 \\
0 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
1 \\
-2
\end{bmatrix}.
\]

using \( B^{-1} \), where \( B = \begin{bmatrix} 0 & 1 & 2 \\ -2 & 0 & 2 \\ 0 & 1 & 1 \end{bmatrix} \) is the given matrix.
5. (15 points) Solve the following differential equation:

\[
\frac{dy}{dt} - 3y = 3y^2, \quad y(0) = 1.
\]
6. (15 points) Find the solution space for the following system

\[
\begin{align*}
x' &= 4x + y \\
y' &= -9x - 2y.
\end{align*}
\]

Also, find the solution to the initial value problem if \(x(0) = 1\) and \(y(0) = 2\).
7. (15 points) Find the general solution of the following differential equation

\[ y'' + 5y' + 4y = 2 \sin 3t. \]
8. \(15\) points) Find the general solution of the following differential equation

\[ y'' - 2y' + y = \frac{e^t}{1 + t^2}. \]