MAT 286   Final exam Fall 2011

Please circle your instructor:

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Instructions

• Please draw a box around your final answer.

• Show and explain all computation. Correct answers with no work will receive no credit.

• Please give your final answer to two decimal places when appropriate.

• Keep your cell phone in your pocket at all times. You may not use it even for telling the time. Any cell phone use will have to be noted, and you may be subject to disciplinary proceedings.

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Total / 100
Question 1. Find the volume of the solid of revolution formed by rotating about the $x$-axis the region bounded by the curves $f(x) = \sqrt{3x - 1}$, $y = 0$, $x = 1$ and $x = 2$.
Question 2. Find the following integrals.

(a) \[ \int_{4}^{9} \frac{1}{x^2} - \frac{1}{\sqrt{x}} \, dx. \] [5 pts]

(b) \[ \int_{2}^{3} \frac{\ln x}{x} \, dx. \] [5 pts]
(c) \[ \int (x + 3) \sqrt{x^2 + 6x} \, dx \] [5 pts]

(d) \[ \int 3x \sin(x) \, dx \] [5 pts]
Question 3. Initially, a tank holds 50 gallons of a mixture of water and dog saliva (25 gallons of this mixture is dog saliva and 25 gallons is water). The tank is stirred constantly. A dog trainer wants to lower the concentration of dog saliva in the tank, so she adds 3 gallons of pure water to the tank every minute, and opens a tap that allows 2 gallons per minute to drain away. How much saliva remains in the tank after 5 minutes? [10 pts]
Question 4. Find the following integrals.

(a) \[ \int_0^2 \int_1^{e^{2x}} \frac{x}{y} \, dy \, dx. \] [10 pts]

(b) \[ \iint_R y e^{xy} \, dy \, dx \text{ where } R \text{ is the region } 1 \leq x \leq 2 \text{ and } 0 \leq y \leq 1. \] [10 pts]
Question 5. (a) Find a specific solution to the initial value problem

\[ \frac{dy}{dx} = ye^{2x} \; ; \; y(1) = 2. \]

[10 pts]

Question 6. Find a general solution to

\[ x \frac{dy}{dx} + y = x^2 + x. \]

[10 pts]
Question 7. Find the improper integral $\int_{1}^{\infty} \frac{1}{3x - 2} \, dx$. [10 pts]

You must show all working. Answers without working will receive no credit.

Question 8. Find the average value of the function $f(x) = e^{2x}$ in the interval $[1, 3]$. [10 pts]