MAT 285
Final Exam, December 15, 2010

Name ____________________________________________________________

Circle your instructor’s name:
Conners, Hsiang, Monachino, Rahmati, Shaw, Wehrli.

PLEASE READ:
1. TURN OFF and PUT AWAY all cell phones and similar devices. Take off all your hats.

2. You may use a calculator on this exam. You may NOT use any calculator with symbolic manipulation capabilities such as the TI89 or TI92. You may NOT share a calculator with another student.

3. Answers without supporting work/reasons will receive no credit. Label answers and give units when appropriate. Please be neat.

4. There are 10 problems (with parts) and total of 11 sheets of papers, printed only on one side, to this exam, including this cover page and a blank sheet, at the end, for scratch work. Be sure that you have all the pages - CHECK NOW!

5. PUT YOUR NAME ON EVERY PAGE.

PLEASE DO NOT WRITE BELOW

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1. Suppose you purchase a new car for $15,000.00. The value of the car decreases by $2,000.00 each year. [10 points]

   (a) Find a \textit{linear function} that models the value (in dollars) of the car as a function of time (in years).

   (b) What is the \textit{value} (in dollars) of your car at the end of the third year from the day of your purchase.

2. Cobalt-60 has a half-life [half-life: the time, for a radioactive substance, it takes for exactly half of the initial mass to decay.] of 5.2 years. The mass of a given sample of Cobalt-60 is 100 grams. [10 points]

   (a) Find the \textit{function} that represents the mass (in grams), of the given sample of Cobalt-60, as a function of time (in years).

   (b) Find the \textit{mass} (in grams), of Cobalt-60, that remains after 20 years.
3. Determine whether the following limits exist. If a limit exists, find its value.

(a) [3 points]
\[ \lim_{x \to 1} \frac{5x^2 - 7x + 2}{x^2 - 1} = \]

(b) [3 points]
\[ \lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25} = \]

(c) [4 points]
\[ \lim_{h \to 0} \frac{(x + h)^2 - x^2}{h} = \]
4. For the following functions, find the indicated derivatives (do NOT simplify):

(a) \( f(x) = x^4 - x^{(-\frac{1}{3})} + e^{(x^5 - 2x)} \), \( \text{[3 points]} \)

\[ f'(x) = \]

(b) \( w(z) = \frac{\ln(z)}{z^3 + 5} \), \( \text{[3 points]} \)

\[ w'(z) = \]

(c) \( s(t) = t \sin(t) \)

[You are asked to compute the second derivative of the function \( s(t) \).] \( \text{[4 points]} \)

\[ s''(t) = \]
5. The position (in meters) of a particle moving along a line is given by:

\[ s(t) = -t^3 + 4t^2 + 8, \]

where \( t \) is in minutes. [10 points]

(a) Find the velocity when \( t = 3.5 \) minutes. Include units in your answer.

(b) Find the acceleration when \( t = 1.5 \) minutes. Include units in your answer.
6. At the balloon stand of a Wegmans Supermarket, the machine pumps 9 cm$^3$/sec of Helium into a spherical balloon. Find the rate of change of the radius of a spherical balloon when the radius is 3 cm. Include units in your answer. [10 points]

(The formula of the volume of a sphere with radius $r$: $V = \frac{4\pi}{3}r^3$)
7. Given that \( f(x) = \frac{1}{3} x^3 + 2x^2 - 5x - \frac{13}{8} \). [Show your work!] [10 points]

(a) Find the critical number(s) of \( f(x) \), then test each critical number for a relative max/min or neither.

(b) Find the open interval(s) on which \( f(x) \) is increasing, and the open interval(s) on which \( f(x) \) is decreasing.

(c) Find the open interval(s) on which the graph of \( f(x) \) is concave up, and the open interval(s) on which the graph of \( f(x) \) is concave down.

(d) Find the inflection point(s), \((x, y)\), [find the \( x \)- and \( y \)-coordinates of the inflection point(s)], on the graph of \( f(x) \).
8. An open box is to be made by cutting a square from each corner of a 18-inch by 18-inch piece of metal and then folding up the sides. What size square should be cut from each corner in order to produce a box of maximum volume? [Show your work and justify that your dimensions maximize the volume.] [10 points]
9. Given that \( x \) and \( y \) are related by the equation \( xy + x^2 - y^2 = 4 \). [10 points]

(a) Using \textit{implicit differentiation}, find \( \frac{dy}{dx} \).

(b) Find \textit{an equation of the tangent line} to the graph of \( xy + x^2 - y^2 = 4 \) at the point \((2, 2)\).
10. Given that \( f(x, y) = \frac{x^3}{3} + \frac{y^2}{2} - xy \) [10 points]

(a) Find all the critical points, find the \( x\)- and \( y\)-coordinates, of \( f(x, y) \).

(b) Classify (relative max/min or saddle point) the critical points found in part (a).