Syracuse University
MAT284-M100 & M200, Business Calculus
Spring 2011
Instructor: Dr. A. Bourhim

Final Exam

• Write your full name.

• Write clearly, and give sufficient details to justify your answers. Any answer without details will be considered wrong.

• The use of mobile phones or pagers is not allowed.

Question 1: /10
Question 2: /10
Question 3: /10
Question 4: /15
Question 5: /15
Question 6: /10
Question 7: /20
Question 8: /10

TOTAL: /100
1. A manufacturer sells his product at $14 per unit, selling all he produces. His fixed cost is $9,810 and his variable cost per unit is $5.

(a) How many units must the manufacturer sold to earn a profit of at most $10,210?

(b) Find the break-even quantity and revenue.
2. Macy’s is selling rain jackets. The total cost per jacket is $110. What should be the original price on the tag of each jacket so that Macy’s can reduce the original price by 20% during a sale and still make a profit of $92.8 per jacket.
3. A radioactive element decays according to the formula

\[ N(t) = 603 \ e^{-0.032t}, \]

where \( N \) is the amount of milligrams present after \( t \) years.

(a) What is the initial amount of such radioactive element?

(b) Determine the half-life of such radioactive element.

(c) In how many years will one third of the initial amount be present.
4. Find each of the following limits.

(a) \( \lim_{x \to -1} \frac{x^2 + 5x + 4}{x^2 - 1} \).

(b) \( \lim_{x \to 3} \frac{x - 3}{\sqrt{x + 1} - 2} \).

(c) \( \lim_{x \to \infty} \frac{2x^4 + 6x^2 - x + 1}{-5x^4 + x - 7} \).
5. Find

(a) \( \frac{dy}{dx} \) if \( y = \ln \left( \sqrt{4x^2 - 4} \right) \).

(b) \( f'(x) \) if \( f(x) = (x^2 - 3x + 1)^9 \).

(c) an equation of the tangent line to the graph of \( f(x) = x^4 - 3x^2 + 4 \) when \( x = 1 \).
6. For a product, let $q = m^2 + \sqrt{m^2 + 19}$ be the quantity produced per day by $m$ workers, and let $p = -0.1q + 80$ be the demand function.

(a) Find $q$ when 9 workers are hired.

(b) Find the marginal revenue $\frac{dr}{dq}$ and evaluate it when 9 workers are hired.

(c) Find $\frac{dq}{dm}$ and evaluate it when $m = 9$.

(d) Find the marginal revenue-product when $m = 9$. 
7. Let

\[ f(x) = \frac{1}{4}x^4 - 2x^2 + 10. \]

(a) Find the domain of \( f \).

(b) Find the derivative of \( f \), and solve the equation \( f'(x) = 0 \).

(c) Determine where \( f \) is increasing and decreasing, and find the relative extrema.
(d) Find the second derivative of $f$, and solve the equation $f''(x) = 0$.

(e) Test the concavity of $f$, and find the inflection points.
For a product, the profit function is

\[ P = -q^3 + 108q^2 + 219q - 6200. \]

Determine the level of production at which the profit is maximal.