1. (10 points) Solve the initial value problem

\[ y' - (4t^3 + 2t)y = 0, \quad y(0) = 1. \]
Is the matrix \( A \) invertible? Justify your answer. (If \( A \) is invertible, you do not need to compute its inverse.)

\[
\begin{bmatrix}
5 & 2 & 1 \\
2 & 3 & 1 \\
0 & 0 & 2 \\
\end{bmatrix} = A
\]
3. (10 points) Find the general solution of the differential equation

\[ y'' - 6y' + 9y = 0. \]
4. (10 points) Consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$,

$$T(x_1, x_2, x_3) = (2x_1 - x_2, x_1 + x_3, x_2 + 2x_3, x_1 + 2x_2 + 5x_3).$$

(i) Find the standard matrix of $T$.

(ii) Find the kernel of $T$ and specify its dimension.
5. (10 points) Consider the linear system of differential equations

\[ x' = 2x + y, \quad y' = 6x - 3y. \]

(i) Find the general solution of the system.

(ii) What type of equilibrium point does this system have at the origin? Justify your answer.
6. (10 points) Find the function $f(t)$ if its Laplace transform is

$$\mathcal{L}\{f(t)\} = \frac{s}{s^2 + 2s + 5}.$$
7.(10 points) Solve the initial value problem

\[ y'' + 3y' + 2y = 4\delta(t - 5), \quad y(0) = y'(0) = 0. \]