Final Exam

April 30th, 2009
Business Calculus
Instructor: Anas Barhoun
MAT 284, Section U001

Name: .............................................
SUID: .............................................

- To receive full credit, you must explain your answers. Answers without supporting work shown on the paper will receive NO credit.

- There are 10 problems and 11 pages including this cover page. Be sure that you have all pages.

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Problem 1. (10 points)
Find the indicated limits:

\[
\lim_{x \to -3} \left( \frac{x^4 - 81}{x^2 + 8x + 15} \right) =
\]

\[
\lim_{x \to -\infty} \left( \frac{x^4 - 81}{x^2 + 8x + 15} \right) =
\]

\[
\lim_{x \to 3} \left( \frac{x^4 - 81}{x^2 + 8x + 15} \right) =
\]

\[
\lim_{x \to \infty} \left( \frac{3 - 4x^2 + 3}{x^2 + 1} \right) =
\]
Problem 2. (10 points)

a) Find an equation of the tangent line to the curve \( y = \frac{5}{1-3x} \) at the point \( x = 2 \). (5 points)

b) Differentiate the following function: (5 points)

\[
y = \ln \left( \frac{\sqrt{5x^2 + 7}}{3x + 4} \right)
\]
Problem 3. (10 points)
The demand equation for a certain product is:

\[ q = \sqrt{2500 - p^2} \]

where \( p \) is given in dollars.

Find the point of elasticity of demand when \( p = 30 \). If the price of 30 decreases \( \frac{2}{3} \% \), what is the approximate change in demand?
Problem 4. (10 points)
a) The marginal-revenue function for a manufacturer’s product is given by
\[ \frac{dr}{dq} = 2000 - 40q - 3q^2 \]
Find the demand function. (5 points)

b) Find the following indefinite integral: (5 points)
\[ \int \left( \frac{e^x}{5} + \frac{\sqrt{x^2}}{3x} \right) \, dx \]
Problem 5. (10 points)
Let consider the following function

\[ f(x) = x^3 - 3x - 9 + 5 \]

a) Determine intervals on which the function is increasing and intervals on which it is decreasing. (3 points)

b) Determine any relative maxima or/and relative minima. (2 points)

c) Determine intervals on which the function is concave up, concave down and determine any inflection point. (3 points)

d) Sketch the graph of the function \( f \). (2 points)
Problem 6. (10 points)
a) Find $x$ and express your answer in terms of natural logarithm: (5 point)

\[ 6e^{2x} - 1 = \frac{1}{2} \]

b) Solve the following equation for $x$. (5 points)

\[ 10^{\log x^2} = 4 \]
Problem 7. (10 points)

a) A company invests a total of $50,000 of surplus funds at two annual rates of interest: 6% and 7.25%. It wishes an annual yield of no less than 7%. What is the least amount of money that the company must invest at the 7.25% rate. (5 point)

b) If the supply and demand equations of a certain product are $120p - q - 240 = 0$ and $100p + q - 1200 = 0$, respectively, find the equilibrium price. (5 points)
Problem 8. (10 points)
A manufacturer determines that \( m \) employees will produce a total of \( q \) units of a certain product per day, where:

\[
q = \frac{100m}{\sqrt{m^2 + 19}}
\]

If the demand equation for the product is:

\[
p = \frac{4500}{q + 10}
\]

a) Determine how many unit would will produced by 9 employees. (2 points)

b) Determine the marginal revenue when \( m = 9 \). (3 points)

c) Determine the marginal quantity with respect to \( m \) when \( m = 9 \). (3 points)

d) Conclude the marginal-revenue product when \( m = 9 \). (2 points)
Problem 9. (10 points)
A manufacturer sells a product at $8.25 per unit, selling all produced. The fixed cost is $2000 and the variable cost is $7.75 per unit.

a) At what level of production will there be a profit of $5000. (3 points)

b) At what level of production will there be a loss of $1250. (3 points)

c) At what level of production will the break-even point occurs. (4 points)
Problem 10. *(10 points)*
For a monopolist, the cost per unit of producing a product is $3, and the demand equation is given by:

$$p = \frac{12}{\sqrt{q}}$$

What price will give the greatest profit?