Please PRINT your name here:

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MAT 284, Final Examination
4 May, 2009

• Check to see that you have 9 pages and 11 questions. No credit will be given for problems from missing pages. Print your name on the top of each question page.

• Do not take the exam apart. Use backs of pages for scratchwork.

• Show your work clearly. You will receive NO CREDIT for answers unsupported by sufficient work, even if your answer is correct.

• Review all your work before turning in your exam.

• The number in parentheses before each question is its point value out of 101 points.

Please do NOT write below this line.

2. [12]
3. [15]
4. [15]
5. [12]
6. [13]
7. [6]
8. [16]
9. [12]

Total
Have you read the instructions on the cover page?

1. (12) Differentiate the following. (Use the rules for derivatives rather than the definition of the derivative. You need not simplify your answers.)

   (a) \( f(x) = x^4 e^{-2x} \)

   (b) \( y = \frac{5t - 4}{t^2 + 7} \)

   (c) \( g(x) = \sqrt{4 + \ln x} \)
2. (9) (a) Give the *general form* for the antiderivatives of the function

\[ f(x) = 3\sqrt{x} - \frac{1}{7x}. \]

(b) Give the *particular* antiderivative whose graph passes through the point \((1, 4)\).

3. (6) Give the equation of the line that is tangent to the graph of \(y = (3x - 2)^4 + 7\) at the point \((1, 8)\).
4. (7) Find the derivative of \( g(r) = \ln \left( \frac{x\sqrt{x} + 5}{2x - 7} \right) \) without using the product rule or the quotient rule.

5. (8) Assume that the marginal cost function for a product is \( \frac{dc}{dq} = 15 + 0.06q \) and that the fixed costs for that product are $7000. Find the average cost function for the product.
6. (12) The demand equation for the product manufactured by a certain monopolist is

\[ p = 600 - 3q. \]

The fixed costs for manufacturing this product are $21,600 and the variable costs are $60 per item.

(a) Give the total cost function.

(b) Express the monopolist's profit as a function of the number \( q \) of items sold.

(c) What value of \( q \) will yield the maximum profit?
7. (8) The supply and demand equations (in some order) for a hotel room (per night) in a convention city are:

\[ p = 100 - 0.02q \quad \text{and} \quad p = 0.04q + 40 \]

(a) Find the equilibrium price \( p = \) ________________

(b) Suppose that a new tax of $3.00 per occupied room per night is to be paid by all hotels and motels. Fill in the new supply and demand equations in the blanks below.

supply equation ______________________

demand equation ______________________

8. (5) A cell phone manufacturer will produce 12,000 cell phones at a price of $12 each. For each $2 increase in the price offered for the phones, the manufacturer will produce an additional 3000 phones. Find the linear equation relating the price \( p \) per phone and the number \( q \) of cell phones produced.
9. (6) (a) If \( f(x) = x^2 e^x \) and \( g(x) = 2x - 5 \), then

\[
(f \circ g)(x) = \quad
\]

(b) Assume \( h(x) = \sqrt{x^2 + 7} \). Find functions \( f(x) \) and \( g(x) \) such that \( h(x) = f(g(x)) \).

Do not use \( f(x) = x \) or \( g(x) = x \).

\[
f(x) = \quad \quad \quad g(x) = \quad
\]
10. (16) If \( f(x) = \frac{1}{4}x^4 - 2x^2 + \frac{5}{2} \), then \( f'(x) = x(x + 2)(x - 2) \). Use this information to answer the following questions. Show your work below.

(a) On what intervals is the function
   increasing? ________________
   decreasing? ________________
   concave up? ________________
   concave down? ________________

(b) List the \( x \)-values of
   the critical points of the function. ________________
   the relative maximum (or maxima) of the function. ________________
   the relative minimum (or minima) of the function. ________________
   the inflection point(s) of the function. ________________

Show your work here:
11. (12) The demand equation for a certain product is \( p = \frac{765}{2q + 5} \), where \( q \) is the quantity of the product produced and sold while \( p \) is the unit price when \( q \) units are produced.

(a) Obtain a function \( \eta(q) \) expressing the elasticity of demand as a function (only) of \( q \).

(b) Evaluate the point elasticity of demand when \( q = 10 \) and determine whether the demand is elastic or inelastic or neither?