Do NOT open this booklet until you are told to do so. You will have 2 hours to complete this exam. Show ALL work required to solve the problems. You may use a non-symbolic calculator on any part of this exam, but you may not share a calculator with another student. Be sure to write down your calculations, even if you use your calculator. You may not use any other electronic devices.

Do NOT write in the space below as this space will be used for grading.

1. ________
2. ________
3. ________
4. ________
5. ________
6. ________
7. ________
8. ________
9. ________
10. ________
11. ________
12. ________

TOTAL ________
(1) [6 pts] Simplify the following expression:

\[
\frac{1}{(x+h)^2} \cdot \frac{1}{x^2} \cdot h
\]

[5 pts] Let \(2^a = 3\) and \(2^b = 7\). Solve for \(x\) in terms of \(a\) and \(b\): \(3^x = 49\)

(2) [5 pts] Let \(f(x) = 2x^2 - 3x\). Find the average rate of change of this function over the interval from \(x = -1\) to \(x = 2\).
(3) [6 pts] Find a formula for the piecewise function shown below:
(4) [10 pts] The town of Taylor had a population of 5000 people in the year 1980 (t = 0).

(a) The town's population grows at a rate of 8% per year. Write a formula for the town's population, P(t), in terms of t, in years.

(b) When did the town reach a population of 15,000 people? Solve algebraically. Round your final answer to the nearest year.

(5) [8 pts] Solve the following equation for x exactly.

\[ \ln(8x) - 5\ln(2x) = -1 \]
(6) [6 pts] The graph of $f$ is shown below. Find formulas in terms of $f$ for the transformations of $f$ shown in graph (a) and graph (b).

Graph (a): $y =$ ____________  

Graph (b): $y =$ ____________
(7) [12 pts] A coffee shop finds that its weekly profit is a function of the price, \( x \), it charges per cup. If \( x \) is in dollars, the weekly profit is \( P(x) = -2600x^2 + 7800x - 2898 \) dollars.

(a) Find algebraically the maximum profit and the price per cup that produces that profit. Show any formulas that you use.

(b) Which function, \( P(x - 2) \) or \( P(x) - 2 \), gives a function that has the same maximum profit? Justify your answer. What price per cup produces that maximum profit?

(c) Which function, \( P(x + 60) \) or \( P(x) + 60 \), gives a function where the price per cup that produces the maximum profit remains unchanged? Justify your answer. What is the maximum profit?
(8) [8 pts] A weight hangs on a string of length 4 feet and swings through 6° on either side of the vertical. How long is the arc through which the weight moves from one high point to the next high point? Give an exact answer. Show any formulas that you use.
(9) [10 pts] A population of rabbits oscillates between a low of 1290 on January 1 \((t = 0)\) and a high of 2238 on July 1 \((t = 6)\).

(a) Find a formula for the population, \(P\), in terms of the time, \(t\), in months.

(b) Sketch the graph of the function. Show the amplitude, period, and midline clearly on your graph. Give the equation of the midline.

(10) [6 pts] Complete the table below given that \(h(x) = f(g(x))\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>(g(x))</th>
<th>(f(x))</th>
<th>(h(x))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>
(11) [10 pts] Let \( f(x) = \frac{x + 6}{3 - x} \). Find the x-intercept(s) and the y-intercept.

Find the equations of the vertical asymptote(s) and the horizontal asymptote(s).

Use this information to draw a sketch of the graph.

Evaluate the following: \( \lim_{{x \to 3^+}} \frac{x + 6}{3 - x} \) and \( \lim_{{x \to -\infty}} \frac{x + 6}{3 - x} \).
(12) [8 pts] Find a possible formula for the polynomial shown below: