Do NOT open this booklet until you are told to do so. You will have 2 hours to complete this exam. **Show ALL work required to solve the problems.** You may use a non-symbolic calculator on any part of this exam, but you may not share a calculator with another student. Be sure to write down your calculations, even if you use your calculator. You may not use any other electronic devices.

Do NOT write in the space below as this space will be used for grading.

1. 
2. 
3. 
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7. 
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9. 
10. 
11. 
12. 

TOTAL
(1) [10 pts] Simplify each of following:

\[ \frac{1}{(x + h)^2} - \frac{1}{x^2} \]

Simplify the following and write without negative exponents:

\[ (2ab)^{-3}(a^3b^{-2})^2 \]

(2) [5 pts] Let \( h(x) = 2x^2 - 5x + 3 \). Find the average rate of change of this function over the interval from \( x=-3 \) to \( x=2 \).
(3) [10 pts] A 3 foot tree is purchased and planted in the spring. After 4 years, it is $5 \frac{1}{2}$ feet tall. Assume that the relationship between the height of the tree and years after planting is linear.

(a) Find an equation to represent the height of the tree for any number of years $t$ after planting.

(b) Approximately how tall will the tree be 8 years after planting?
(4) [10 pts] (a) Find a formula for a parabola with zeroes at $x=-3$ and $x=5$ and which passes through the point $(2,5)$. Include a sketch of the graph of the parabola.

(b) Find the coordinates of the vertex of this parabola algebraically.
In the year 1995, town of Greenburg had a population of 1800 people. Its population has been declining by 3% each year. Answer all questions to the nearest hundredth of a unit. Show all calculations!

(a) Find a formula for \( P(t) \), the population \( t \) years after 1995.

(b) What will be the population of the town in the year 2015?

(c) In what year does the population reach 750 people?
(6) [10 pts] Solve each of the following equations exactly.

\[ 5e^{3x^2} = 40 \]

\[ \log(7 - x) - \log(4 + x) = 2 \]

(7) [6 pts] Given the graph of \( y = f(x) \) below, sketch the graph of \( y = f(-x) + 2 \). Be sure to show the images of each of the labeled points on the graph.
(8) [6 pts] Match the limit statements below with one the graphs of the functions I - IV. If there is no match, write "none."

\[ \begin{align*}
(a) & \quad \lim_{x \to 0^-} f(x) = \infty \\
(b) & \quad \lim_{x \to 0^+} f(x) = 0 \\
(c) & \quad \lim_{x \to 2^-} f(x) = -\infty \\
(d) & \quad \lim_{x \to \infty} f(x) = 0 \\
(e) & \quad \lim_{x \to \infty} f(x) = 2 \\
(f) & \quad \lim_{x \to \infty} f(x) = \infty \\
\end{align*} \]

Graph # _____

Graph # _____

Graph # _____

Graph # _____

Graph # _____

Graph # _____

Graph # _____

Graph # _____
(9) [12 pts] A ferris wheel is 28 meters in diameter and boarded in the six o'clock position from a platform that is 3 meters above the ground. The wheel completes one full revolution every 16 minutes. At $t=0$, you are in the six o'clock position and ascending.

Find a formula for your height above the ground after $t$ minutes on the ferris wheel.

Sketch a graph of this function for one complete revolution. Label the amplitude, period, and horizontal shift. Show the midline and give its equation. Label each axis and show the scale.
(10) [5 pts] Let \( H(x) = \sqrt{1 - x^2} \). Find \( f(x) \) and \( g(x) \) such that \( H(x) = f(g(x)) \). Do not let \( f(x) = x \) or \( g(x) = x \).

[5 pts] The functions \( f(x) \) and \( g(x) \) are given by the following tables:

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>1</td>
<td>3</td>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(x) )</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Find \( f(g(5)) \) ________

Find \( g(f(6)) \) ________
(11) [10 pts] Let \( f(x) = \frac{x-1}{x^2-16} \). Find the x-intercepts, the y-intercept, and all horizontal and vertical asymptotes. Show all work! Include a sketch of the graph. Label each asymptote with its equation.

Evaluate \( \lim_{x \to \infty} f(x) \).
(12) [10 pts] Find a possible formula for this function: