MAT 183  Final Exam  Dec. 16, 2009
Ver. 1

Signature:

Instructions: Write the answers and show the main steps of your work on this test sheet.

There are 17 questions on 16 pages (including this cover).

Be sure you have all 16 pages (8 sheets) and that they are all of the same version.

You should spend no more than 8 minutes on each problem; be sure that you get to the easier parts of each problem.

The Final Exam is scored on a basis of 100 points and will count 25% of your final grade.

You must show your work to get full credit! If you use the calculator, write down your input.

DO NOT WRITE ON THE REST OF THIS COVER SHEET!

Problem 1 (6)  Problem 6 (4)  Problem 12 (5)
Problem 2 (6)  Problem 7 (6)  Problem 13 (6)
Problem 3 (6)  Problem 8 (6)  Problem 14 (6)
Problem 4 (9)  Problem 9 (6)  Problem 15 (6)
Problem 5 (7)  Problem 10 (6)  Problem 16 (6)
Problem 11 (5)  Problem 17 (4)

Test 1 Total (34)  Test 2 Total (33)  Test 3 Total (33)

EXAM TOTAL (100)
Problem 1. (6 points) Solve completely each of the following systems of equations. Write out the values of the variables if the system has a unique solution; write "No solution" if the system has no solution; write two of the variables in terms of the third in the case that there are many solutions.

(i) \[
\begin{align*}
2x - 4y & = 6 \\
-x + 2y - 5z & = -4 \\
2x - 4y + 10z & = 2
\end{align*}
\]

(ii) \[
\begin{align*}
x - 4y & = 6 \\
-2x + 2y - 5z & = -4 \\
2x - 4y + 10z & = 2
\end{align*}
\]
Problem 2. (6 points) Katie is buying fish for her aquarium. Goldfish cost $8 each, angelfish are $7 each, and catfish are $5 dollars each. Katie wants to have twice as many goldfish as angelfish and catfish combined. She wants to spend a total of $108 and buy 42 fish all together. How many of each kind of fish should she buy?

(i) (2 points) Identify the variables. Use $g$, $a$ and $c$ and explain in words EXACTLY what each variable represents:

$g$,  
$a$,  
$c$,  

(ii) (4 points) Give the system of equations that you plan to use.

(iii) (2 points) Solve the system and interpret your answer.
Problem 3. (6 points)

(i) (3 points) A corporation has an electronics division and an industrial equipment division. For each $1 worth of output, the electronics division needs $0.06 worth of electronics and $0.10 worth of equipment. For each $1 worth of output, the industrial equipment division needs $0.02 worth of electronics and $0.05 worth of equipment. The current demand is for $1,630,000 worth of electronics and $2,500,000 worth of industrial equipment. Set up the input-output matrix and demand matrix. Do NOT solve.

(ii) (3 points) A simplified economy has the input-output matrix, $A$, and the demand matrix, $D$, given below. Find at what level the divisions need to produce to meet demand.

$$A = \begin{bmatrix} 0.05 & 0.08 \\ 0.02 & 0.06 \end{bmatrix} \quad D = \begin{bmatrix} 918,000 \\ 450,300 \end{bmatrix}$$
Problem 4. You want to make a 5 letter word out of the letters A, B, C, D and E.

(i) (3 points) How many different 5 letter words are possible if repetitions are allowed?

(ii) (3 points) How many different 5 letter words are possible if repetitions are not allowed?

(iii) (3 points) How many different 5 letter words begin with a B if repetitions are not allowed?
Problem 5. (7 points) A survey of 100 college freshmen who exercise regularly found that 45 jog, 30 swim, 20 cycle, 6 jog and swim, 1 jogs and cycles, 5 swim and cycle, and 1 does all three.

(i) (2 points) In the following Venn Diagram, shade the region that represents those students that jog and swim, but do not cycle.

(ii) (5 points) Compute the number of students who do not jog, swim or cycle. Show your work by either giving the formula that you use or by filling in the number of students in each region of the following Venn Diagram.
Problem 6. (4 points) In a certain class 32% of the voters are republicans, 31% democrats, 24% independent and the remaining voters belong to the green party. A random voter is selected.

(i) What is the probability that he or she is not a democrat?

(ii) What is the probability that he or she is a member of the green party?

Problem 7. (6 points) The probability of event $A$ occurring is $\frac{1}{3}$; the probability of event $B$ occurring is $\frac{1}{2}$; the events $A$ and $B$ are independent; the probability of event $C$ occurring is $\frac{1}{4}$; finally, the probability of both events $B$ and $C$ occurring is $\frac{1}{8}$.

(i) What is the probability that event $A$ and $B$ occur?

(ii) What is the probability that event $B$ or $C$ occur?

(iii) What is the probability that event $B$ occurred given that event $C$ occurred?
Problem 8. (6 points) You have a box containing 2 white markers and 3 black markers; you also have a white urn containing 4 orange markers and 3 blue markers and a black urn containing 2 orange markers and 5 blue markers.

You select one marker from the box. If it is white, select a marker from the white urn; if it is black, select a marker from the black urn.

(i) (5 points) Make a tree diagram of this experiment. Be sure to include labels for all edges and for the end of each path!

(ii) (1 point) Circle the most likely color for the second marker you choose:

orange blue
Problem 9. (6 points) Consider the following probability distribution for the random variable $X$

<table>
<thead>
<tr>
<th>$k$</th>
<th>$P(X = k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
</tr>
</tbody>
</table>

(i) (3 points) Compute the expected value of $X$ (Show your work or your calculator entry)

$$E(X) = _____$$

(ii) (3 points) Compute the standard deviation of $X$ (Show your work or your calculator entry)

$$\sigma(X) = _____$$
Problem 10. (6 points) A factory produces lightbulbs, and 3% of them are defective.

(i) (4 points) Given a random sample of 50 lightbulbs, find the mean and standard deviation for the number of defective lightbulbs in the sample.

$$\mu = \underline{}$$

$$\sigma = \underline{}$$

(ii) (2 points) Given a random sample of 50 lightbulbs, what is the probability that exactly 4 of them are defective?

Problem 11. (5 points) The scores on a certain test are normally distributed with mean 73 and standard deviation 10 as pictured below. What percentage of the students scored between 55 and 70?

YOU MUST SHOW YOUR WORK!

[If you use the calculator, write out exactly what you enter; if you use the table show your calculations.]
Problem 12. (5 points) Consider the following absorbing stochastic matrix:

\[
A = \begin{bmatrix}
1 & 0 & 0.3 & 0 \\
0 & 1 & 0.1 & 0 \\
0 & 0 & 0.5 & 0.4 \\
0 & 0 & 0.1 & 0.6
\end{bmatrix}
\]

(i) (3 point) Identify the absorbing states of \( A \).

(ii) (2 points) Determine the fundamental matrix for \( A \).
Problem 13. (6 points) For a certain group of states, it was observed that 50% of Democratic governors were succeeded by Democrats and 50% by Republicans. Also, 40% of the Republican governors were succeeded by Democrats and 60% by Republicans.

(i) (3 points) Write a stochastic matrix describing the transitions in this problem.

(ii) (3 points) Suppose 35% of the current governors are Democrats and 65% are Republicans. Assuming that the current trend holds, what percent of the governors will be Democrats after the next election?
Problem 14. (6 points) Consider the following investments at 3.6% APR, compounded monthly. To get credit you must show how you computed your answer. If you use the TVM Solver, fill in the table in the left hand margin otherwise write out the formula that you use.

(i) (2 points) If you invest $20,000 today, what will this investment be worth at the end of 8 years?

(ii) (2 points) Approximately how many months would it take for an investment of $20,000 to grow to $30,000?

(iii) (2 points) How much should you invest today if you want to have $20,000 in 5 years?
Problem 15. (6 points) Consider a savings account paying 3.6% interest compounded monthly into which you make regular deposits at the end of each month. To get credit you must show how you computed your answer. If you use the TVM Solver fill in the table in the left hand margin otherwise write out the formula that you use.

(i) (2 points) If your payments are $1,540 each, how much will be in your account at the end of 5 years?

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<tbody>
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<td></td>
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<tr>
<td>PV=</td>
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<td>PMT=</td>
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<tr>
<td>FV=</td>
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<td>P/Y=</td>
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<tr>
<td>C/Y=</td>
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</tbody>
</table>

(ii) (2 points) How much of this is interest that you have earned? Circle one:

$8,672.67 $10,741.70 $13,948.55 $15,599.93 NONE OF THESE

(iii) (2 points) How much should you deposit today if you want to withdraw $1,200 at the end of each month for the next 5 years?

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<td>P/Y=</td>
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<td>C/Y=</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
Problem 16. (6 points) Consider a $220,000, 25 year mortgage at 3.6\% interest compounded monthly. To get credit you must show how you computed your answer.

(i) (2 points) Compute your monthly payments:

\[
\begin{align*}
N &= \\
I\% &= \\
PV &= \\
PMT &= \\
FV &= \\
P/Y &= \\
C/Y &= \\
\end{align*}
\]

(ii) (2 points) How much of your first payment is interest? You must show your computations!

(iii) (2 points) If the bank charges you 2 points, how much of the loan actually goes toward the purchase of the house? Circle one:

$\begin{array}{c} 
\$222,200 \\
\$220,000 \\
\$217800 \\
\$215600 \\
NONE\ OF\ THESE
\end{array}$
Problem 17. (4 points)

(i) You borrow $12,000 at 8% for two years with payments computed by the add-on method. Your monthly payments are? Circle one:

\$580.00  \$676.67  \$773.33  \$870.00  NONE OF THESE

(ii) You are planning to set up an IRA for your retirement. You expect to be in the 26% tax bracket until you retire and you expect to be in the 23% bracket throughout your retirement. What type of IRA should you set up? Circle one:

TRADITIONAL IRA  ROTH IRA  IT MAKES NO DIFFERENCE
Probability and Statistical formulas:

(i) \( Pr(E') = 1 - Pr(E) \)

(ii) \( Pr(E \cup F) = Pr(E) + Pr(F) - Pr(E \cap F) \)

(iii) \( Pr(E \cap F) = Pr(E) \times Pr(F) \) if and only if \( E \) and \( F \) are independent

(iv) \( Pr(E \cap F) = 0 \) if and only if \( E \) and \( F \) are mutually exclusive

(v) \( Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)} \)

(vi) \( \mu = x_1p_1 + \cdots + x_np_n \)

(vii) \( \sigma = \sqrt{(x_1 - \mu)^2p_1 + \cdots + (x_n - \mu)^2p_n} = \sqrt{x_1^2p_1 + \cdots + x_n^2p_n - \mu^2} \)

(viii) For the binomial random variable \( X \) with \( N \) trials and probability \( p \) of success:

\[
Pr(X = k) = C_k p^k (1-p)^{N-k}
\]

\[
\mu = Np \quad \text{and} \quad \sigma = \sqrt{Np(1-p)}
\]

(ix) \( Z = \frac{X - \mu}{\sigma} \)

Finance formulas

Simple interest: \( F = (1 + yr)P \).

Effective interest: \( 1 + e = (1 + \frac{r}{c})^c \),

where \( c \) is the number of compounding periods per year.

Periodic rate: \( i = \frac{r}{c} \),

where \( c \) is the number of compounding periods per year.

Compound interest: \( B_k = (1 + i)B_{k-1} \) giving \( F = (1 + i)^nP \)

Increasing annuity: \( B_k = (1 + i)B_{k-1} + R \) giving \( F = \frac{(1+i)^n-1}{i}R \)

Decreasing annuity: \( B_k = (1 + i)B_{k-1} - R \) giving \( P = \frac{(1+i)^n-1}{i(1+i)^n}R \)
### TABLE 1: Areas under the standard normal curve

<table>
<thead>
<tr>
<th>$z$</th>
<th>$A(z)$</th>
<th>$z$</th>
<th>$A(z)$</th>
<th>$z$</th>
<th>$A(z)$</th>
<th>$z$</th>
<th>$A(z)$</th>
<th>$z$</th>
<th>$A(z)$</th>
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<tbody>
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<td>0.0002</td>
<td>-2.00</td>
<td>0.0228</td>
<td>-0.50</td>
<td>0.3085</td>
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<td>0.0256</td>
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<td>0.8531</td>
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