**Signature:**

**Instructions:** Write the answers and show the main steps of your work on this test sheet.

There are 14 questions on 15 pages (including this cover and a formula sheet).

Be sure you have all 15 pages (8 sheets) and that they are all of the same version.

You should spend no more than 8 minutes on each problem; be sure that you get to the easier parts of each problem.

The Final Exam is scored on a basis of 100 points and will count 25% of your final grade.

You must show your work to get full credit! If you use the calculator, write down your input.

**DO NOT WRITE ON THE REST OF THIS COVER SHEET!**

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**EXAM TOTAL (100)**
Problem 1. (5 points) In each case below, a system of three equations in $w$, $x$, $y$, and $z$ (in that order) has been put in row-reduced echelon form. You are to interpret this matrix. In each case circle one of “No Solution”, “Unique Solution” or “Many Solutions.” If you circle “Unique Solution”, fill in the values of $w$, $x$, $y$, and $z$. If you circle “Many Solutions”, fill in the values for 3 of the variables in terms of the 4th.

(i) \[
\begin{bmatrix}
1 & 0 & 0 & 0 & 4 \\
0 & 1 & -1 & 0 & -3 \\
0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

- No Solution
- Many Solutions
- Unique Solution

For Unique Solution, fill in the values of $w$, $x$, and $z$.

(ii) \[
\begin{bmatrix}
1 & 0 & 0 & 0 & 4 \\
0 & 1 & 0 & 0 & -3 \\
0 & 0 & 1 & -1 & 1 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

- No Solution
- Many Solutions
- Unique Solution

For Unique Solution, fill in the values of $w$, $x$, and $z$.

(iii) \[
\begin{bmatrix}
1 & 0 & 0 & 0 & 4 \\
0 & 1 & 0 & 0 & -3 \\
0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1
\end{bmatrix}
\]

- No Solution
- Many Solutions
- Unique Solution

For Unique Solution, fill in the values of $w$, $x$, and $z$. 
Problem 2. (8 points) You are selling boxes of a mixture of 3 types of donuts. Box 1 contains 6 jelly donuts and 6 chocolate donuts and costs $8.10; Box 2 contains 6 jelly donuts 3 chocolate donuts and 3 plain donuts and costs $7.80 and Box 3 contains 4 of each type and costs $7.40. What is the cost of a single donut of each type?

(i) (2 points) Identify the variables. Use \( j \), \( c \), and \( p \) and explain in words EXACTLY what each variable represents:

\( j \),

\( c \),

\( p \).

(ii) (6 points) Give the system of equations that you plan to use, but DO NOT SOLVE THE SYSTEM!
Problem 3. (6 points)

(i) (3 points) A corporation has a plastics division and an industrial equipment division. For each $1$ worth of output, the plastics division needs $0.04$ worth of plastics and $0.08$ worth of equipment. For each $1$ worth of output, the industrial equipment division needs $0.01$ worth of plastics and $0.05$ worth of equipment. The current demand is for $1,532,100$ worth of plastics and $2,976,100$ worth of industrial equipment. Set up the input-output matrix and demand matrix. Do NOT solve.

(ii) (3 points) Given the input-output matrix, $A$, and the demand matrix, $D$, below, find at what level the divisions need to produce to meet demand.

$$A = \begin{bmatrix} 0.04 & 0.09 \\ 0.02 & 0.07 \end{bmatrix} \quad D = \begin{bmatrix} 918,120 \\ 476,370 \end{bmatrix}$$
Problem 4. (9 points) A box has 8 slips of paper with the numbers 0, 1, 2, 3, 4, 5, 6, and 7 written on them.

(i) (3 points) How many different outcomes are possible if you select 4 slips one at a time without replacing the slips you select?

(ii) (3 points) How many different outcomes are possible if you select 4 slips one at a time replacing each slip before you select the next slip?

(iii) (3 points) How many different outcomes are possible if you select 4 slips at the same time?
Problem 5. (6 points) A school of 710 students offers Math, English and Social Studies. There are 400 students in Math, 425 in English and 350 in Social Studies; 200 students are taking both Math and English, 200 are taking both Math and Social Studies and 200 are taking both Social studies and English. Finally 70 students are taking all three subjects.

(i) (1 point) In the following Venn Diagram, shade the region that represents those students taking Math and English but not Social Studies.

(ii) (5 points) Compute the number of students in the school taking none of these courses. Show your work by either giving the formula that you use or by filling in the number of students in each region of the following Venn Diagram.
Problem 6. (7 points) You have a box containing 2 white markers and 3 black markers; you also have a white urn containing 4 orange markers and 8 blue markers and a black urn containing 2 orange markers and 5 blue markers.

You select one marker from the box. If it is white, select a marker from the white urn; if it is black, select a marker from the black urn.

(i) (6 points) Make a tree diagram of this experiment. Be sure to include labels for all edges and for the end of each path!

(ii) (1 point) Circle the most likely color for the second marker you choose:

orange     blue
Problem 7. (6 points) Consider the following probability distribution for the random variable $X$.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$P(X = k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0.3</td>
</tr>
</tbody>
</table>

(i) (3 points) Compute the expected value of $X$ (Show your work or your calculator entry)

\[ E(X) = \] 

(ii) (3 points) Compute the standard deviation of $X$ (Show your work or your calculator entry)

\[ \sigma(X) = \]
Problem 8. (8 points) Assume that you are given a probability space with events $E$, $F$, and $G$. Suppose that $P(E) = 0.5$, $P(F) = 0.6$, $P(G) = 0.3$, $P(F \text{ and } G) = 0.2$, that $E$ and $F$ are independent, and that $E$ and $G$ are mutually exclusive. Compute the following probabilities:

(i) (2 points) $P(E')$

(ii) (2 points) $P(E \cap F)$

(iii) (2 points) $P(F \cup G)$

(iv) (2 points) $P(G|F)$
Problem 9. (6 points) Compute the following probabilities

(i) (2 points) What is the probability that you will get exactly 5 questions correct if you are guessing on a 12 question multiple choice test where each question has 4 choices.

(ii) Given that 68% of the voters in a given county are registered as independents
   (a) (2 points) How many independents would you expect to find in a random sample of 100 voters from that county? [Give the formula that you use.]
   (b) (2 points) What is the standard deviation of the distribution of the number of independents in a random sample of 100 voters from that county? [Show your work]
Problem 10. (6 points) The scores on a certain test are normally distributed with mean 73 and standard deviation 10 as pictured below. 
YOU MUST SHOW YOUR WORK!
[If you use the calculator, write out exactly what you enter; if you use the table show your calculations.]

(i) (4 points) What percentage of the students scored between 62 and 72.

(ii) (2 points) What is the probability that a randomly selected student scored 81 or more on the test?
Problem 11. (9 points) Consider the following absorbing stochastic matrix:

\[
A = \begin{bmatrix}
1 & 0 & 0.3 & 0.1 \\
0 & 1 & 0.4 & 0.1 \\
0 & 0 & 0.2 & 0.3 \\
0 & 0 & 0.1 & 0.5 \\
\end{bmatrix}
\]

(i) (1 point) Identify the absorbing states of \( A \).

(ii) (4 points) Determine the stable matrix for \( A \).

(iii) (2 points) What is the probability of eventually being in absorbing state II if you begin in non-absorbing state III?

(iv) (2 points) What is the expected number of iterations that you will be in states III or IV before entering an absorbing state, if you begin in state IV?
Problem 12. (8 points) Consider the following investments at 7.2% APR, compounded monthly. To get credit you must show how you computed your answer. If you use the TVM Solver, fill in the table in the left hand margin otherwise write out the formula that you use.

(i) (3 points) If you invest $12,000 today, what will this investment be worth at the end of 5 years?

<table>
<thead>
<tr>
<th>N</th>
<th>I%</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
<th>P/Y</th>
<th>C/Y</th>
</tr>
</thead>
</table>

(ii) (2 points) Approximately how many months would it take for an investment of $12,000 to grow to $20,000?

<table>
<thead>
<tr>
<th>N</th>
<th>I%</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
<th>P/Y</th>
<th>C/Y</th>
</tr>
</thead>
</table>

(iii) (3 points) How much should you invest today if you want to have $12,000 in 5 years?

| N | I% | PV | PMT | FV | P/Y | C/Y |
Problem 13. (8 points) Consider a savings account paying 7.3% interest compounded monthly into which you make regular deposits at the end of each month. To get credit you must show how you computed your answer. If you use the TVM Solver fill in the table in the left hand margin otherwise write out the formula that you use.

(i) (3 points) If your payments are $540 each, how much will be in your account at the end of 5 years?

| N= |
| I%= |
| PV= |
| PMT= |
| FV= |
| P/Y= |
| C/Y= |

(ii) (2 points) How much of this is interest that you have earned?

| N= |
| I%= |
| PV= |
| PMT= |
| FV= |
| P/Y= |
| C/Y= |

(iii) (3 points) How much should you deposit each month if you want to have $12,000 in 5 years?

| N= |
| I%= |
| PV= |
| PMT= |
| FV= |
| P/Y= |
| C/Y= |
Problem 14. (8 points) Consider a $220,000, 25 year mortgage at 7.2% interest compounded monthly. To get credit you must show how you computed your answer.

(i) (2 points) Compute your monthly payments:

\[
\begin{array}{l}
N= \\
I\%= \\
PV= \\
PMT= \\
FV= \\
P/Y= \\
C/Y=
\end{array}
\]

(ii) (3 points) How much of your first payment is interest?

\[
\begin{array}{l}
N= \\
I\%= \\
PV= \\
PMT= \\
FV= \\
P/Y= \\
C/Y=
\end{array}
\]

(iii) (3 points) Suppose that you can only afford monthly payments of $1,400. How large a 25 year mortgage at 7.3% can you afford?

\[
\begin{array}{l}
N= \\
I\%= \\
PV= \\
PMT= \\
FV= \\
P/Y= \\
C/Y=
\end{array}
\]
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