MAT 296 FINAL EXAM

NAME:             Student ID:

Instructor: Madsen Murphy Omninon Purin Watkins Yerrington (Circle one)

Write your answers and show all your work on this test. There are 9 problems on 12 pages (including this cover sheet), for a total of 200 points. To receive full or partial credit, the correct work leading to the correct answer must be written down. Unsupported answers will receive little or no credit. Graphics calculators may be used. Symbolic calculators, such as TI-89 or TI-92, may not be used.

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1. (30 points) Consider the region $A$ enclosed by the curves

$$y = \frac{1}{x}, \quad y = \sqrt{x}, \quad x = 1 \quad \text{and} \quad x = 2.$$  

(a) Find the exact area of $A$.

(b) Set up an integral that gives the volume of the solid obtained by rotating the region $A$ about the $x$-axis. Do not evaluate the integral.

(c) Set up an integral that gives the volume of the solid obtained by rotating the region $A$ about the line $x = 1$. Do not evaluate the integral.
2. (10 points) A bacteria culture starts with 1000 bacteria and grows at a rate proportional to its size. After 2 hours there are 9000 bacteria. Find the number of bacteria after 3 hours.
3. (20 points) **Set up**, but do not evaluate, each of the following integrals that gives 
(a) the length of the parametric curve

\[ x = \ln t, \quad y = \sqrt{t + 1}, \quad 1 \leq t \leq 5. \]

(b) the area of the surface obtained by rotating the curve \( y = \sqrt{x}, \ 1 \leq x \leq 8, \) about the \( y \)-axis.
4. (15 points) Consider the polar curve \( r = \theta^2, \ 0 \leq \theta \leq 2\pi \).

(a) Sketch this curve. (As always a “sketch” should include axis-labels and labels for all important points.)

(b) Find the area of the region that is bounded by this curve and lies in the sector where \( 0 \leq \theta \leq 1 \).
5. (40 points) Compute each of the following integrals:

(a) \[ \int t \sin(2t) \, dt \]

(b) \[ \int \frac{-4x - 16}{x^3 + 16x} \, dx \]
(c) \[ \int \sin \theta \tan^2 \theta \, d\theta \]

(d) \[ \int_1^3 \frac{1}{\sqrt{3-x}} \, dx \]
6. (10 points) Find the total area of the infinitely many circles on the interval $[0, 1]$ in Figure 1.
7. (30 points) For each of the following series, decide whether it is convergent or divergent. Be sure to justify your answer by indicating which test for convergence is used.

(a) \[ \sum_{n=1}^{\infty} \frac{1}{n + \sqrt{n}} \]

(b) \[ \sum_{n=2}^{\infty} \frac{n}{\ln(n^3)} \]
(c) \[ \sum_{n=1}^{\infty} \frac{e^n}{n!} \]
8. (25 points) Find the radius and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n(x - 3)^n}{\sqrt{n4^n}}.$$

Specify for each $x$ in the interval of convergence whether the power series is absolutely convergent or conditionally convergent. Justify your answer in complete detail.
9. (20 points) Consider the function \( f(x) = xe^{-x^2} \).

(a) Find the Maclaurin series of \( f(x) \) and its interval of convergence.

(b) Using the Maclaurin series from (a), evaluate \( \int_0^1 xe^{-x^2} \, dx \) to within an error of 0.01. Use the fewest number of terms possible to attain the prescribed accuracy.