Name: __________________________________________
Signature: _______________________________________

Instructor: (please circle one)

P. CARNEY K. DIVERIS J. ENTNER V. FATICA
D. GUSTAFASON A. MCCAFFERY L. SHEN M. VENOUZIOU

Read before you begin:

1. This exam contains 10 problems for a total of 150 points. There are 13 pages including the cover sheet. You are responsible for checking that they are all present.

2. You must show all work to completely justify your answers in order to receive any credit. Unsupported answers will receive little or no credit.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>20</td>
</tr>
<tr>
<td>2.</td>
<td>15</td>
</tr>
<tr>
<td>3.</td>
<td>10</td>
</tr>
<tr>
<td>4.</td>
<td>20</td>
</tr>
<tr>
<td>5.</td>
<td>10</td>
</tr>
<tr>
<td>6.</td>
<td>15</td>
</tr>
<tr>
<td>7.</td>
<td>15</td>
</tr>
<tr>
<td>8.</td>
<td>15</td>
</tr>
<tr>
<td>9.</td>
<td>10</td>
</tr>
<tr>
<td>10.</td>
<td>20</td>
</tr>
<tr>
<td>Total</td>
<td>150</td>
</tr>
</tbody>
</table>
1. (20 points) Find the following limits, if they exist. **SHOW ALL WORK.**

(a) \( \lim_{x \to 0} \left( \frac{1}{x} - \frac{1}{|x|} \right) \)

(b) \( \lim_{x \to +\infty} \frac{x \sin \left( \frac{5}{x} \right)}{1 + \tan^2 \left( \frac{1}{x} \right)} \)
(c) \( \lim_{x \to 0} \frac{e^x - 1 - x}{x^2} \)

(d) \( \lim_{x \to \infty} (1 - \frac{2}{x})^x \)
2. (15 points) Find the derivative of each of the following functions. **DO NOT SIMPLIFY YOUR ANSWER. SHOW ALL WORK.**

(a) \( f(x) = \sin^{-1}(\cos x) + x^\pi + \pi^x \)

(b) \( f(x) = \sin^4(2\pi x) + e^{2x} \cos(3x) \)

(c) \( f(x) = \frac{(x - 2)^3 \sqrt{x^2 + 1}}{(x + 3)^{2/3} e^{x^2}} \)  
   (hint: use logarithmic differentiation)
3. (10 points) Use implicit differentiation to find the equation of the line tangent to the graph of the ellipse \( \frac{x^2}{25} + \frac{y^2}{100} = 1 \) at the point (3, 8).
4. Let $f$ be a function $f(x) = 2x^3 - 3x^2 - 12x + 7$ defined on the closed interval $[0, 3]$.

(a) *(10 points)* Find the absolute maximum and minimum values of $f$ on the closed interval $[0, 3]$.

(b) *(5 points)* By using the Intermediate Value Theorem, show that there is a root of the equation $f(x) = 0$ in the interval $[0, 3]$.

(c) *(5 points)* Find a number $c$ in the interval $[0, 3]$ that satisfies the conclusion of the Mean Value Theorem for the function $f$. 
5. \textit{(10 points)} Use the limit definition of the derivative to find $f'(a)$, if $f(x) = \frac{2}{\sqrt{x+3}}$, $a > -3$. Show all work using the limit laws. \textbf{NO CREDIT} if the definition is not used.
6. (15 points) At the same time, one car starts traveling north at 50 miles per hour and another car starts traveling east at 65 miles per hour. Two hours later, how fast is the distance between the cars changing?
7. (15 points) A cylindrical can without a top is made to contain 1000 cm$^3$ of liquid. Find the dimensions that will minimize the surface area of the metal to make the can.
8. (15 points) Consider the function \( f(x) = \frac{x^2}{x^2 + 3} \).

(a) Find all vertical and horizontal asymptotes of \( f \), if any exist. \textit{Justify your answer.}

(b) Find all open intervals on which \( f \) is increasing and those on which \( f \) is decreasing. \textit{Justify your answer.}

(c) Find all open intervals on which \( g \) is concave up, and those on which it is concave down. \textit{Justify your answer.}
9. (a) (5 points) Let \( f(t) = \int_0^{t^{1/2}} x(2 + x^5)^{1/2} \, dx \) be the position of an object at time \( t \).

Find the velocity of the object at time \( t = 5 \).

(b) (5 points) Let \( g(x) = \begin{cases} 
5, & x < 1 \\
-2x + 2, & x \geq 1 
\end{cases} \). Compute \( \int_{-3}^{8} g(x) \, dx \).
10. (20 points) Evaluate the following definite and indefinite integrals.

(a) \( \int \left( \frac{1-x}{x} \right)^2 \, dx \)

(b) \( \int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx \)
(c) $\int_0^2 x^2 \sqrt{1 + x^3} \, dx$

(d) $\int_0^4 |\sqrt{x} - 1| \, dx$