Final Exam
Math 284-100 Fall 2008
Meyer

Show your work. You may use a calculator, but you must write the steps you take with the calculator. A correct answer with no work will not receive credit. Point values are indicated.

1. (10 points) Let \( a = \ln 2, b = \ln 5 \) and \( c = \ln 11 \). Write

\[
\ln \left( \frac{11}{50} \right)
\]

in terms of \( a, b \) and \( c \).

1. ________________

2. (10 points) The demand equation of a certain company’s product is given by \( p = 155 - 2.8q \), where \( p \) is the price in dollars of their product when demand is \( q \) units. Find the revenue generated when demand is 12 units.

2. ________________
3. (10 points) The supply and demand curves for a particular product are, respectively,

\[ p = 13 + 0.4q \quad \text{and} \quad p = 31 - 0.1q. \]

Find the equilibrium price \( p \) and quantity \( q \).
4. (10 points) Find the following limits. Circle your answers.

a) \[ \lim_{x \to 2} \frac{x^2 + 2x}{x^2 + 4x + 3} \]

b) \[ \lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 7x + 12} \]

5. (10 points) Look at the graph of the function below. Find the indicated limits and function values. Write “doesn’t exist” if the value or limit does not exist.

a) \[ \lim_{x \to 1} f(x) = \] ____________

b) \[ f(-1) = \] ____________

c) \[ f(3) = \] ____________

d) \[ \lim_{x \to 3^-} f(x) = \] ____________

e) \[ \lim_{x \to 3^+} f(x) = \] ____________
6. (10 points) Use the definition of the derivative to find $f'(x)$ for

$$f(x) = 2x^2 + x.$$ 

You must use the definition of the derivative to receive any credit for this problem. Circle your answer.
7. (25 points) Find the derivative of each function. Use appropriate notation. Circle your answer. You do not need to simplify your answers.

(i) \( f(x) = x^4 - 5x^3 - 2x + 3 \)

(ii) \( y = 3x + \frac{1}{\sqrt{x}} \)

(iii) \( g(t) = t^3 e^{2t} \)

(iv) \( f(x) = \frac{x^2 - 1}{x^2 + x} \)

(v) \( g(x) = \ln (x^3 + 3) \)
8. (10 points) Find an equation of the line tangent to the curve

\[ y = (3x - 2)^2 + 4 \]

at \( x = 1 \).
9. (10 points) For a particular product, \( m \) workers produce \( q \) items where

\[
q = 3m + \frac{100}{m}.
\]

The demand equation for the product is

\[
p = 200 - 2q.
\]

Find the marginal revenue product, \( \frac{dr}{dm} \), when \( m = 10 \).

(Remember that you need to find the revenue function and use the relationship

\[
\frac{dr}{dm} = \frac{dr}{dq} \cdot \frac{dq}{dm}.
\]

9. ____________________
10. (10 points) Use calculus to sketch the graph of the function

\[ f(x) = x^3 - 9x^2 + 15x - 6. \]

Indicate on a number line intervals upon which the function is increasing and intervals upon which the function is decreasing.

Indicate on a different number line intervals upon which the function is concave up and intervals upon which the function is concave down.

On the graph, label with coordinates any relative extremes and inflection points. Sketch the graph on the set of axes provided.

Be sure to show your work. Simply graphing the function from your calculator and doing some calculus will not be sufficient to receive credit.
11. (10 points) The revenue function for a particular product is \( r = 3210q - 12q^2 \). The total cost function is \( c = 5100 + 18q \). Find the production level, \( q \), and corresponding price, \( p \), that give the maximum profit.
12. (10 points) Find the function $y$ such that

$$\frac{dy}{dx} = 4x^3 - 8x$$

and $y(1) = 5$. 

12. ______________________