MAT 183  Spring 2008 Final Exam

Name__________________________________
(please print)

Signature________________________________

Circle your instructor's name: Banerjee Doerr

There are 13 questions on this exam. Answer all questions! Show all work! Little or no credit will be given for unsupported answers.

You MUST show all formulas that you use!

Show all values when using TVM Solver and syntax when using TVM variables.

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<tr>
<th>Problem</th>
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<td>Total</td>
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(1) [5 pts] The matrix $A$ below shows the number of packages shipped by three departments via overnight delivery (ON), second day delivery (2nd), and regular delivery (Reg). Matrix $B$ shows cost of shipping by UPS and FedEx.

$$
\begin{array}{cc}
\text{dept} & \text{ON} & \text{2nd} & \text{Reg} \\
1 & 3 & 8 & 7 \\
2 & 5 & 4 & 3 \\
3 & 1 & 0 & 5 \\
\end{array}
\begin{array}{cc}
\text{UPS} & \text{FedEx} \\
\text{ON} & \begin{bmatrix} 11.50 & 12.50 \end{bmatrix} \\
\text{2nd} & \begin{bmatrix} 9.50 & 8.50 \end{bmatrix} \\
\text{Reg} & \begin{bmatrix} 3.50 & 4.00 \end{bmatrix} \\
\end{array}
$$

Compute $AB$.

Interpret the meaning of the entry in the second row and first column.
(2) [8 pts] Tickets Online has a package deal for three concerts: the Rock Stars, the Smooth Sounds and the Baroque Band. A customer must buy a ticket to all three concerts to get the special prices, shown in the chart below.

<table>
<thead>
<tr>
<th></th>
<th>High school students</th>
<th>College students</th>
<th>Adults</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock Stars</td>
<td>$6</td>
<td>$9</td>
<td>$10</td>
</tr>
<tr>
<td>Smooth Sounds</td>
<td>$4</td>
<td>$6</td>
<td>$8</td>
</tr>
<tr>
<td>Baroque Band</td>
<td>$3</td>
<td>$5</td>
<td>$6</td>
</tr>
</tbody>
</table>

The total ticket sales were $16760 for the Rock Stars, $12040 for the Smooth Sounds and $9440 for the Baroque Band.

(a) Write a system of equations that can be used to determine the number of tickets sold to high school students, college students, and adults. You must state what quantities your variables represent.

(b) Solve the system of equations.
(3) [8 pts] The Rockford Corporation has a digital electronics division and a plastics division. For each dollar's worth of plastic produced, the plastics division uses $0.05 work of plastic and $0.10 worth of electronics. For each dollar's worth of electronic equipment produced, the electronics division uses $0.25 worth of plastic and $0.15 worth of electronics.

Show all formulas that you use in solving this problem!

(a) What is the Input-Output matrix for this economy?

(b) Find the value of plastics and electronics that must be produced for the corporation to meet an external sales demand of $38 million worth of plastics and $41 million worth of electronics. Round your answer to the nearest million.
(4) [8 pts] A total of 80 students were surveyed about their musical preferences. The survey found that 25 students liked classical music, 26 liked country music, and 22 liked jazz. There were 5 students who liked country and classical, 7 students who liked country and jazz, and 8 who liked classical and jazz. There were 3 students who liked all three types of music.

Draw a Venn diagram that shows these musical preferences.

How many students like jazz but not classical music?

How many students like only country music?

How many students did not like any of the three types of music?
(5) [9 pts]

(a) How many four-letter passwords can be formed from the letters \{A, B, C, J, K, L, X, Z\} if repetitions are allowed?

(b) How many four-letter passwords can be formed from the letters \{A, B, C, J, K, L, X, Z\} if repetitions are not allowed?

(c) Blackhawk Tech gives 6 presidential scholarships. If there are 40 nominees, in how many ways can the scholarships be awarded?
(6) [9 pts] A tray of electronics equipment contains 25 components, including 6 that are defective. Four components are elected at random.

(a) What is the probability that all four of the components are defective?

(b) What is the probability that exactly three of components are defective?

(c) What is the probability that at least one component is defective?
(7) [10 pts]  A university cafeteria surveyed the students who ate breakfast there for their coffee preferences. The findings are summarized as follows:

<table>
<thead>
<tr>
<th></th>
<th>Do not drink coffee</th>
<th>Prefer regular coffee</th>
<th>Prefer decaf coffee</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>22</td>
<td>141</td>
<td>72</td>
</tr>
<tr>
<td>Male</td>
<td>19</td>
<td>189</td>
<td>57</td>
</tr>
</tbody>
</table>

A student is selected at random from this group.

(a) What is the probability that the student does not drink coffee?

(b) What is the probability that the student is female?

(c) What is the probability that the student is male and prefers regular coffee?

(d) What is the probability that the student is female, given that the student prefers decaf coffee?

(e) What is the probability that the student does not drink coffee, given that the student is male?
(8) [5 pts] The most popular color for sports cars is silver, with 30% of owners preferring that color. If nine sports car owners are selected at random, what is the probability that exactly three of them prefer silver?

(9) [8 pts] A car rental agency has rental and return facilities at both LaGuardia and JFK airports in the New York City area. If a car is rented at either airport, it must be returned to one or the other airport.

Of the cars rented at LaGuardia, 75% are returned to LaGuardia. Of the cars rented at JFK, 85% are returned there. Assume that the company rents all of its cars each day and that each car is rented and returned only once each day. At the beginning of one particular day, there are 250 cars at LaGuardia and 150 cars at JFK.

(a) Write the stochastic matrix representing these transitions. Label the rows and columns.

(b) What is the expected number of cars at each airport the next day?

(c) What is the expected number of cars at each airport 5 days later?
(10) [10 pts] The students at Lakewood College can buy a full-meal plan at the college cafeteria or a one-meal-per-day plan. Some students choose not to buy a meal plan. Each semester, a student may remain in a plan or change to another plan or no plan at all. The percentage of students who change each semester are shown in this transition matrix:

\[
\begin{bmatrix}
\text{full} & \text{one} & \text{none} \\
\text{full} & 0.70 & 0.15 & 0.10 \\
\text{one} & 0.25 & 0.75 & 0.10 \\
\text{none} & 0.05 & 0.10 & 0.80
\end{bmatrix}
\]

Assume that these percentages remain valid over a long period of time.

(a) Set up and solve a system of equations to find the stable distribution.

(b) What percentage of students will have each plan in the long run?
(11) [5 pts] David wants to save money in order to have $11,400 at the end of 8 years. He has an account that pays 5.1% interest compounded quarterly.

(a) How much money should he deposit at the end of each quarter?

(b) At the end of eight years, how much interest will he have earned?

(12) [5 pts] A business loan of $250,000 is to be paid off in monthly payments for 7 years with a $50,000 balloon payment at the end of the seventh year. The interest rate on the loan is 6.5% compounded monthly. Calculate the monthly payment.
(13) [10 pts] You want to purchase an automobile for $25,600. The dealer offers you 0% financing for 36 months or a $3000 rebate. You can obtain 6.2% interest compounded monthly for 36 months from your local credit union.

(a) What is your monthly payment at 0% financing?

(b) Assuming you choose the rebate, what is your monthly payment to the credit union?

(c) Assuming you choose the rebate, write out an amortization schedule for the first two months of the credit union loan.

(d) Assuming you choose the better option, how much will you save over the life of the loan?
Summary of Finance Formulas

Compound Interest

\[ F = P \cdot (1 + i)^n \]
\[ P = F \cdot (1 + i)^{-n} \]

Annuities and Sinking Funds: Future Value

\[ F = R \cdot \left[ \frac{(1 + i)^n - 1}{i} \right] \]
\[ R = F \cdot \frac{i}{(1 + i)^n - 1} \]

Annuities: Present Value

\[ P = R \cdot \left[ \frac{1 - (1 + i)^{-n}}{i} \right] \]
\[ R = P \cdot \left[ \frac{i}{1 - (1 + i)^{-n}} \right] \]