Problem 1. Let $E \subset \mathbb{R}$ with $m(E) > 0$ (i.e. $E$ has positive Lebesgue measure). Show that the set $E - E = \{x - y \mid x, y \in E\}$ contains an interval centered at 0.

Problem 2. Let $\mu$ be a positive measure on $X$ and $f$ measurable on $X$. For $0 < r < p < s < \infty$ show that $\|f\|_p \leq \max(\|f\|_r, \|f\|_s)$.

Problem 3. Prove that a positive measure $\mu$ on $X$ is $\sigma$-finite if and only if there is an $f \in L^1(d\mu)$ with $f(x) > 0$ for all $x \in X$.

Problem 4. Let $1 < p < \infty$ and suppose that $f_k \to f$ in $L^p(\mathbb{R}, m)$ as $k \to \infty$ ($m$ is Lebesgue measure on $\mathbb{R}$). In addition assume that $g_k(x) = \begin{cases} 0, & x < k \\ 1, & x \geq k \end{cases}$ for $k = 1, 2, \ldots$ What does the sequence $f_k g_k$ converge to in $L^p$? Prove it.