MAT 295 FINAL EXAM  
December 12, 2007

Name: _______________________

Signature: _____________________

Instructor: (Please circle one)

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Read before you begin:

1. This exam has 12 problems worth a total of 200 points. There are 8 two-sided pages including this cover sheet. You are responsible for checking that you have a complete test.

2. For an answer to receive full credit or be considered for partial credit, you must show your work. Answers with no supporting work will receive little or no credit.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
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<tr>
<td>5</td>
<td>20</td>
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<td>6</td>
<td>10</td>
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<td>20</td>
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<td>8</td>
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</tr>
<tr>
<td>12</td>
<td>18</td>
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</tbody>
</table>

Total  200
1. (18 points) Use elementary methods to find the following limits if they exist. If a limit does not exist, say so. *Justify your answers completely.* On this problem you may NOT use L'Hospital's Rule because it is not elementary.

(a) \( \lim_{x \to \infty} \frac{2x^3 - 3x^2 + 1}{4x^3 - 2x^2 + x} \).

(b) \( \lim_{x \to 1^+} \frac{x^2}{x - 1} \).
2. (20 points) Find the derivatives of the functions below. **DO NOT SIMPLIFY YOUR ANSWERS. SHOW ALL WORK.**

(a) \( f(x) = \frac{x^3}{\cot x} \)
(b) \( g(t) = \sin(2t^3 - 3) \)

(c) \( h(x) = \arctan e^x \) (Note: Sometimes \( \arctan(e^x) \) is written as \( \tan^{-1}(e^x) \).)

(d) \( f(x) = x^{\csc x} \)
3. (14 points) Consider the curve defined by the equation
\[ x \tan y + 4 \sin y + x^2 = 1. \]

(a) Find \( \frac{dy}{dx} \).

(b) The equation of the line tangent to the curve at the point \((1, 0)\).
4. (10 points) Let $f(x)$ be a function.

(a) Complete the following sentence: “The derivative of $f$ is the function $f'(x)$ defined by $f'(x) = \ldots$”

(b) Use the definition of the derivative to find $f'(x)$ when $f(x) = x^2$. Show all work using the limit laws. If you do not use the definition of the derivative, you will receive NO CREDIT.
5. (20 points) A spotlight on the ground shines on a wall 40 ft away. A man 6 ft tall walks from the spotlight toward the wall at a speed of 6 ft/sec. How fast is the length of his shadow on the wall decreasing when he is 15 ft away from the spotlight?
6. (10 points) Suppose \( f(x) = 3x^2 - x + 2 \) for \( x \) in the closed interval \([1, 4]\).

(a) Show that \( f \) satisfies the hypotheses of the Mean Value Theorem.

(b) Find all numbers \( c \) in the interval \([1, 4]\) that satisfy the conclusion of the Mean Value Theorem.
7. (20 points) Let \( f(x) = 2x^3 - 9x^2 + 12x - 5 \). Use the techniques of calculus to find

(a) The intervals on which \( f \) is increasing and those on which \( f \) is decreasing. Show enough work to justify your answer.

(b) The \((x, y)\) coordinates of all local maxima and local minima of \( f \). Show enough work to justify your answer.
(c) The intervals on which $f$ is concave up and those on which $f$ is concave down. 
*Show enough work to justify your answer.*

(d) The $(x, y)$ coordinates of all points of inflection of $f$. *Show enough work to justify your answer.*
8. (20 points) On the graph paper below sketch the graph of a function $f$ that has the following properties. Label the local maxima, local minima and the points of inflection of $f$.

(a) The domain of $f$ is all $x \neq 1$.
(b) $f(0) = 0$ and $f(-1) = 2$.
(c) $f$ is continuous and twice differentiable on its entire domain.
(d) $\lim_{x \to -1} f(x) = -\infty$
(e) $f$ is decreasing on $(-1, 1)$ and increasing on $(-\infty, -1)$ and $(1, \infty)$.
(f) $f$ is concave down on $(-2, 1)$ and $(1, \infty)$ and concave up on $(-\infty, -2)$.
(g) $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = 1$. 


9. (14 points) Evaluate the following limits:

(a) \( \lim_{x \to \infty} \frac{x^2}{e^x} \).

(b) \( \lim_{x \to 0^+} (\sqrt{x})^x \).
10. (18 points) A box with a square base and top must have a volume of 125 in$^3$. Find the dimensions of the box that minimizes the amount of material used.
11. (18 points) Evaluate the following definite integrals:

(a) \( \int_1^3 12x^3 - 2x^2 + 5x^{-2} \, dx \).

(b) \( \int_1^2 \frac{6x^2 + 3}{2x} \, dx \).
(c) $\int_{0}^{1/2} e^x - \frac{1}{\sqrt{1-x^2}} \, dx$.

12. (18 points) Find the following indefinite integrals:

(a) $\int \csc(2x) \cot(2x) \, dx$. 
(b) \[ \int e^{\cos x} \sin x \, dx. \]

(c) \[ \int x \sec^2 (x^2 + 3) \, dx. \]