Name: __________________  
Signature: ______________  

Instructions: Show all the work you want graded on this exam. Unsupported answers may receive little or no credit. List the values you put into your calculator.

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Part I: Linear Algebra

1. a. (2 pts) Find the sum of the matrices.
\[
\begin{pmatrix}
3 & -6 & -2 & 4 \\
-2 & 7 & 8 & -5
\end{pmatrix}
\]

b. (4 pts) Suppose \(a\) and \(b\) are constants and the inverse of the matrix

\[
\begin{pmatrix}
a & b & 3 \\
1 & 1 & 5 \\
2 & 1 & 9
\end{pmatrix}
\]

is

\[
\begin{pmatrix}
4 & 3 & -3 \\
1 & 3 & -2 \\
-1 & -1 & 1
\end{pmatrix}
\]

Use this to find the solution to the following system of equations:

\[
\begin{cases}
ax + by + 3z = 7 \\
x + y + 5z = 2 \\
2x + y + 9z = 3
\end{cases}
\]
2. Each of the following matrices is the matrix of a system of linear equations in $x$, $y$ and $z$. Find all solutions to the system of equations in each case.

$$
\begin{bmatrix}
1 & 2 & -1 & 3 \\
0 & 0 & 1 & 4 \\
2 & 4 & -2 & 6
\end{bmatrix}
$$

a. (3 pts) $\begin{bmatrix}
1 & 0 & 2 & 2 \\
0 & 1 & 1 & 4 \\
2 & 1 & 5 & 9
\end{bmatrix}$

3. (5 pts) A business has a plastics division and a metal division. For each $\$1$ worth of output, the plastics division needs $10\$ of output from the plastics division and $6\$ from the metal division. For each $\$1$ worth of output, the metal division needs $12\$ of output from the plastics division and $14\$ from the metal division. What level of production should each division have in order to have $\$15$ million worth of plastics and $\$10$ million worth of metal to deliver to market?

Part II: Probability and Statistics

4. (6 pts) Let $A$ and $B$ be events with $\Pr(A) = 0.4$, $\Pr(B) = 0.7$ and $\Pr(A \cup B) = 0.8$. Compute the following:

a. $\Pr(A \cap B)$

b. $\Pr(A | B)$

c. $\Pr(A \cap B')$

d. $\Pr(B' | A)$
5. A class has 10 students, 4 of whom are first year students. An experiment consists of randomly selecting students one at a time without replacement until either three students have been chosen or a first year student has been chosen.

a. (3 pts) Draw a tree diagram to illustrate this experiment. Include probabilities along the branches.

b. (2 pts) What is the probability that the experiment ends by getting a first year student on the second selection?

6. a. (3 pts) A true false test has four questions. Find the probability of getting at least three questions correct by guessing.

b. (3 pts) The lifetimes of a certain model of television’s picture tubes are normally distributed with $\mu = 48$ months and $\sigma = 8$ months. The manufacturer wants to issue a warranty that will be written so that about 92% of the picture tubes will outlast the warranty. For how many months should the picture tubes be guaranteed?
Part III: Math of Finance

7. a. (2 pts) How much time is required for $100 to double if deposited at 6% interest compounded quarterly?

b. In an election, Mary received 92,000 votes. On each recount she receives 2000 votes plus 98% of the number of votes she received on the previous recount.

   (i) (2 pts) If $y_n$ is the number of votes Mary receives on the $n^{th}$ recount, give a difference equation for $y_n$.

   (ii) (2 pts) How many votes will she have after many recounts?

8. (5 pts) John saves for 10 years. He deposits $100 in his savings account at the end of each week for 4 years and then increases his weekly savings to $150 for the remaining 6 years. How much money will he have at the end of the ten years. Assume he earns 10% interest compounded weekly.

9. Jane borrows $250,000 at 8% interest compounded monthly to start a new business.

   a. (3 pts) If the loan is to be paid off in monthly payments for 20 years, how much will her monthly payment be?

   b. (3 pts) After 5 years of payments, her business is struggling and she arranges to pay the remaining balance over 25 years at 8.5% compounded monthly. What will her new monthly payment be?