MAT285– Final Exam

NAME (PRINTED)__________________________________________

TOTAL SCORE: 100      YOUR SCORE__________________

INSTRUCTIONS:

1. PLEASE CIRCLE YOUR INSTRUCTOR: Jinjia Li : Moises Venouziou : Nihat Gogus

2. YOU MUST SHOW ALL OF YOUR WORK ON THE EXAM PAGES. UNSUPPORTED WORK WILL NOT RECEIVE ANY CREDIT.

3. GOOD LUCK!
1. (5pts) Find the domain of the function $f(x) = \sqrt{9 - x^2}$.

2. (5pts) Find $\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$.

3. (5pts) Compute the FIRST derivative and the SECOND derivative with respect to $x$ of $f(x) = x^3 + 8$. 
4. (5pts) Compute the derivative with respect to $x$ of $f(x) = x^2 \sin(3x)$.

5. (5pts) Compute the derivative with respect to $x$ of $f(x) = \frac{e^x}{2x}$.

6. (5pts) Compute the derivative with respect to $x$ of $f(x) = \ln(\sqrt{x} + \epsilon)$. 
7. (6pts) Compute \( \frac{\partial}{\partial y}(x^2 + \tan(xy)) \)

8. (7pts) Use implicit differentiation to compute \( \frac{dy}{dx} \) if \( x \) and \( y \) satisfy the equation

\[ x^3 y^2 - 4x^2 = 1 \]
9. (7pts) Find the equation of the tangent line to the curve \( y = 2^x - 1 \) at the point \((0, 0)\).

10. (7pts) In the year of 3000, the population in the Mars will be 5 million. Assume the population will be growing exponentially with growth constant 0.1. In what year will the population in the Mars reach 10 million?
11. (7pts) Use the limit definition of derivatives to compute \( f'(5) \) for \( f(x) = 15 - x^2 \).
(No partial credit will be given if you use any other method)
12. (8pts) Let \( f(x) = x^3 - 6x^2 - 15x - 5 \). Identify the intervals on which \( f(x) \) is increasing and on which the function is decreasing, identify the intervals on which \( f(x) \) is concave up and on which \( f(x) \) is concave down.
13. (10pts) Let \( f(x) = \frac{1}{3}x^3 + 3x^2 + 5x \). Find all possible relative minimum or relative maximum points and use the Second Derivative Test to determine whether \( f(x) \) has a relative min/max at these points.
14. (10pts) A sheet of cardboard 3 ft. by 4 ft. will be made into a box by cutting equal-sized squares from each corner and folding up the four edges. What will be the dimensions of the box with largest volume?
15. (8pts) A ladder 10 ft long rests against a vertical wall. If the top of the ladder slides down the wall at a rate of 0.75 ft per second. How fast is the bottom of the ladder sliding away from the wall when the top of the ladder is 8 ft above the ground?