MAT 183  Final Exam  Dec. 13, 2006  Ver. A

Signature:

Instructions: Write the answers and show the main steps of your work on this test sheet.

There are 15 questions on 16 pages (including this cover).

Be sure you have all 16 pages (8 sheets) and that they are all of the same version.

You should spend no more than 7 minutes on each problem; be sure that you get to the easier parts of each problem.

The Final Exam is scored on a basis of 100 points and will count 25% of your final grade.

Where indicated, you must show your work to get full credit!

DO NOT WRITE ON THE REST OF THIS COVER SHEET!

Problem 1  Problem 7  Problem 12
Problem 2  Problem 8  Problem 13
Problem 3  Problem 9  Problem 14
Problem 4  Problem 10  Problem 15
Problem 5  Problem 11  Test 3 Total
Problem 6  Test 2 Total
Test 1 Total

EXAM TOTAL:
Problem 1 (4 points) In each case below, a system of three equations in \( x \), \( y \) and \( z \) has been put in row-reduced echelon form. You are to interpret this matrix. In each case circle one of “No Solution”, “Unique Solution” or “Many Solutions.” If you circle “Unique Solution”, fill in the values of \( x \), \( y \) and \( z \).

(Order will be switched on different versions)

a. \[
\begin{bmatrix}
1 & 0 & 1 & | & 0 \\
0 & 1 & -1 & | & -2 \\
0 & 0 & 0 & | & 0
\end{bmatrix}
\]

No Solution Many Solutions Unique Solution \( x = \) \\
\( y = \) \\
\( z = \)

b. \[
\begin{bmatrix}
1 & 0 & 1 & | & 0 \\
0 & 1 & -1 & | & -2 \\
0 & 0 & 0 & | & 1
\end{bmatrix}
\]

No Solution Many Solutions Unique Solution \( x = \) \\
\( y = \) \\
\( z = \)

c. \[
\begin{bmatrix}
1 & 0 & 0 & | & 0 \\
0 & 1 & 0 & | & -2 \\
0 & 0 & 1 & | & 1
\end{bmatrix}
\]

No Solution Many Solutions Unique Solution \( x = \) \\
\( y = \) \\
\( z = \)
Problem 2 (7 points) A bank wishes to invest a $250,000 trust fund in three sources: bonds paying 8% annually, CDs paying 7% annually and mortgages paying 10% annually. The bank wishes to realize an annual income of $21,000 from this investment. A condition of the trust is that exactly $\frac{3}{5}$ of the fund must be invested in bonds and CDs.

How much should the bank invest in each category? Let $b$ denote the amount invested in bonds; $c$ the amount invested in CDs and $m$ the amount invested in mortgages.

a. (4 points) Give the system of equations that you plan to use.

b. (3 points) Solve that system:

\[
b = \\
c = \\
m = \\
\]
Problem 3 (7 points) Consider an industry with 3 sectors, E (energy), I (iron ore) and T (transportation), represented by the following input-output matrix:

\[ [A] = \begin{bmatrix}
E & I & T \\
0.02 & 0.15 & 0.38 \\
0.04 & 0.05 & 0.07 \\
0.01 & 0.15 & 0.03 \\
\end{bmatrix} \]

At what production levels in billions of dollars should each sector operate to meet a demand of $5 billion from sector E, $2 billion from sector I and $6 billion from sector T?

a. (4 points) Write this problem as a matrix equation.

b. (3 points) Fill in your answer (rounded off to two decimal places)

- \( E = \$\)\,______ billion.
- \( I = \$\)\,______ billion.
- \( T = \$\)\,______ billion.
Problem 4 (8 points) A small computer store sells two types of computers, one for home use (H) and one for office use (O). The matrix \( A \) below represents the number of units of memory (M), USB ports (U) and external display ports (E) used on each type of computer.

\[
\begin{bmatrix}
M & U & E \\
2 & 3 & 1 \\
4 & 4 & 2
\end{bmatrix}
\]

The matrix B represents the average daily sales of each type of computer; the matrix \( C \) represents the costs (in dollars) of each type of component:

\[
B = \begin{bmatrix}
H & O \\
2 & 1
\end{bmatrix},
C = \begin{bmatrix}
50 \\
15 \\
75
\end{bmatrix}
\]

a. Compute the following matrix product and explain in words the meaning of each entry in the resulting matrix.

\[
BA =
\]

b. Each computer costs $500 plus the added costs of memory and ports. Give the matrix product that represents the additional cost of memory and ports for each type of computer and compute the total cost for each type of computer.
Problem 5 (6 points) Consider 5-character license plates. The characters can be digits (0, ..., 9) or letters (A, ..., Z). For example: “7X33W”, “AS123”, “RRRRR” and “06040”. In each of the following parts, you must show the basic steps of your work. For example “by the multiplication rule 3 × 4 × 7 = 84” or “we are choosing 4 from 17, 17C4 = 2380”

a. How many license plates have only letters?

b. How many license plates have only digits and no repeated digits?

c. How many license plates have 3 A’s and no other letters?
Problem 6 (4 points) A school of 700 students offers Math, English and Social Studies. There are 400 students in Math, 425 in English and 350 in Social Studies; 200 students are taking both Math and English, 200 are taking both Math and Social Studies and 200 are taking both Social studies and English. Finally 70 students are taking all three subjects.

a. (1 point) In the following Venn Diagram, shade the region that represents those students taking Math and English but not Social Studies.

b. (3 points) Compute the number of students in the school taking none of these courses. Show your work by either giving the formula that you use or by filling in the number of students in each region of the following Venn Diagram.
Problem 7 (8 points) Suppose that there is a test for component X in human blood. If the blood of a person has component X, the test will detect the component 99% of the time; but, 1% of the time it will indicate that the component is not present. On the other hand, if a person’s blood does not have component X, the test will indicate this only 95% of the time and falsely indicate that component X is present 5% of the time. It is known that 74% of the population actually have component X in their blood.

a. (6 points) Make a tree diagram of this testing procedure. Be sure to include all labels!

b. (2 points) Compute the percentage of cases in which a person is told that they have component X in their blood when, in fact, they do not.
Problem 8 (6 points) Consider the following probability distribution for the random variable $X$

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<tr>
<th>$k$</th>
<th>$P(X = k)$</th>
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</thead>
<tbody>
<tr>
<td>0</td>
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<tr>
<td>1</td>
<td>.5</td>
</tr>
<tr>
<td>2</td>
<td>.3</td>
</tr>
</tbody>
</table>

a. Compute the expected value of $X$ (Show your work or your calculator entry)

$E(X) = ______$

b. Compute the standard deviation of $X$ (Show your work or your calculator entry)

$\sigma(X) = ______$
A2 Appendices

![Area under standard normal curve]

**TABLE 1 Areas under the standard normal curve**

<table>
<thead>
<tr>
<th>$z$</th>
<th>$A(z)$</th>
<th>$z$</th>
<th>$A(z)$</th>
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</table>
Problem 9 (7 points) The scores on a certain test are normally distributed with mean 74 and standard deviation 11.3 as pictured below. Letter grades are assigned as follows: F: 0-60; D: 60-70; C: 70-80; B: 80-90; A: 90-100.

a. (1 point) On the graph shade the area that represents the percentage of test scores corresponding to a A.

b. (3 points) What percentage of the test scores correspond to a B? YOU MUST EXPLAIN HOW YOU GOT YOUR ANSWER! [If you use the calculator, write out exactly what you entered; if you use the table show your calculations.]

c. (3 points) What cutoff for an A must you choose if you want about 15% of your students to get an A? YOU MUST EXPLAIN HOW YOU GOT YOUR ANSWER! [If you use the calculator, write out exactly what you entered; if you use the table show your calculations.]
Problem 10  (6 points) A recent poll of residents in a certain community revealed the following information about voting preferences:

<table>
<thead>
<tr>
<th></th>
<th>Democrat</th>
<th>Republican</th>
<th>Independent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>450</td>
<td>400</td>
<td>350</td>
</tr>
<tr>
<td>Female</td>
<td>600</td>
<td>300</td>
<td>400</td>
</tr>
</tbody>
</table>

A person is selected at random from this community.

a. What is the probability that the person is a male?

b. What is the probability that the person is a female independent?

c. What is the probability that the person is a republican given that he is a male?

Problem 11  (5 points) In one state's daily lottery there is 25% probability that any lottery ticket will win something. Suppose that you buy a ticket each day for 15 days and let X denote the number of days that you win.

a. (1 points) The distribution of the random variable X is (circle one)

   Binomial    Normal    None of These

b. (4 points) What is the probability that you will win exactly 4 times out of the 15? To get credit you must show how you computed your answer.
Problem 12 (6 points) Consider investing $10,000 for 5 years at 7.2% interest compounded monthly. To get credit you must show how you computed your answer. For example: "TVM Solver with $N$=, $I$%=, etc." or write out the formula that you use.

a. What is the investment worth at the end of the 5 years?

$$F = \ldots$$

b. How many months would it take for your investment to grow to $15,000?

$$N = \ldots$$

c. How much should you invest today if you want $15,000 at the end of 5 years?

$$P = \ldots$$
Problem 13  (8 points) Consider saving $500 a month for 5 years in an account paying 7.2% interest compounded monthly. Deposits are made at the end of each month. To get credit you must show how you computed your answer. For example: "TVM Solver with N=, I% =, etc." or write out the formula that you use.

a. (2 points) How much is in your account at the end of the 5 years?

b. (3 points) How much of this is interest that you have earned?

c. (3 points) Give the difference equation and initial value that describes the balance at the end of each month in terms of the balance at the end of the previous month.
Problem 14 (8 points) Consider a $200,000, 25 year mortgage at 7.2% interest compounded monthly. To get credit you must show how you computed your answer. For example: "TVM Solver with N=, I% =, etc." or write out the formula that you use.

a. (2 points) Compute your monthly payments:

\[ R = \\]

b. (3 points) How much of your first payment is interest?

c. (3 points) Suppose that you can only afford payments of $1,300. How large a 25 year mortgage at 7.2% can you afford?
Problem 15 (10 points) You wish to buy a $21,000 car. What is the total amount (including interest) that you will pay for the car with each of the following schemes? Show your work!

a. (2 points) Take a 5 year loan at the special rate of 6.5% interest, compounded monthly, for the full amount of the car.

b. (4 points) Take a 10% price reduction and a 5 year loan for the reduced price at the rate of 11% interest, compounded monthly.

c. (4 points) Save $390 a month for a year at 6.5% interest, compounded monthly. Use this amount as a down payment and take a 4 year loan at 11% interest, compounded monthly, for the remaining amount.