MATH 183
FINAL FALL 2003

PART I

1. (a) How many different unordered selections of 3 newspapers can be made from a set of 10 newspapers?

(b) Given a selection of 3, how many ways can the remaining 7 be ordered in a single stack?

2. An urn contains 6 red balls and 4 green balls. A sample of 7 balls is selected at random. What is the probability that exactly 4 red balls are selected?
3. Urn I contains 3 red balls and 1 white ball. Urn II contains 2 red and 2 white balls. Select an urn at random. Then select a ball at random.

(a) Draw the tree diagram for this experiment.

(b) If a red ball is selected, what is the probability that Urn I was chosen?

4. A die is tossed 6 times.

(a) Compute the probability that a "one" appears exactly 4 times.

(b) Compute the expected number of "ones".
PART II

5. A normally distributed random variable $X$ has a probability distribution with a mean of 20 and a standard deviation of 3. Compute $\Pr \{16 < X < 24\}$.

6. How much should be deposited at the end of each month into an account that earns 6% compounded monthly in order to have a balance of $9,000 after 4 years?

7. A loan of $9,000 is to be repaid over 4 years at 12% interest compounded monthly.

   (a) Calculate the monthly payment

   (b) Calculate the balance after 2 years.
8. (a) Solve the difference equation

\[ y_n = 3 \ y_{n-1} + 2. \]

(b) If \( y_0 = 10 \) find \( y_3 \).

(c) Does \( y_n \) increase without bound, decrease without bound, oscillate, increase to a fixed (stable) value, or decrease to a fixed (stable) value?
PART III

9. Given the augmented matrix below, perform the first elementary row operation that should be used in the Gaussian elimination method in order to put the coefficient matrix into diagonal form.

\[
\begin{array}{ccc|c}
1 & 0 & 3 & 9 \\
0 & 1 & -3 & 2 \\
0 & -5 & 4 & 1 \\
\end{array}
\]

10. Solve the system. If there is no solution, state so. If there are infinitely many solutions find two.

\[
x - y - 2z = 2 \\
y - 2z = 1 \\
-3x - 2y - 8z = 7
\]
11. Let \( A = \begin{bmatrix} 3 & 1 \\ -1 & 0 \end{bmatrix} \) and \( B = \begin{bmatrix} 2 & 7 \\ 6 & 4 \\ 1 & 6 \end{bmatrix} \). Compute \( AB \) if defined and compute \( BA \) if defined. If a product is not defined, state so.

12. An economy consists of steel and coal. The steel industry consumes $0.25$ of coal and $0.02$ of steel to produce $1.00$ of steel. The coal industry requires $0.04$ coal and $0.01$ steel to produce $1.00$ coal.

(a) Write the input-output matrix.

(b) How much steel and coal should be produced in order to meet a demand of $1.93$ million steel and $2.38$ million coal?
### Table 1: Areas under the standard normal curve

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