This examination has 10 problems. It is your responsibility to make sure that all are present.

Show ALL work. Minimal credit will be given for answers without supporting work.

A graphics calculator may be used on this examination. However, a symbolic calculator, such as the TI-89 or TI-92, may NOT be used.

Do Not Write Below

1. _______
2. _______
3. _______
4. _______
5. _______
6. _______
7. _______
8. _______
9. _______
10. _______

TOTAL _____
(1) [15 pts] Determine each of the following:

(a) \( \lim_{x \to 0} \frac{e^x - 1}{\ln(x + 1)} \)

(b) the limit of the sequence \( a_n = \left( 1 - \frac{2}{n} \right)^{3n} \)
(2) [10 pts] Consider the region in the first quadrant bounded by \( y = x^2 \) and \( y = x \).

(a) Draw a carefully labeled diagram and set up an integral to find the volume when this region is rotated about the x-axis. Do NOT evaluate the integral.

(b) Draw a carefully labeled diagram and set up an integral to find the volume when this region is rotated about the y-axis. Do NOT evaluate the integral.
(3) [15 pts] A tank has the shape of a paraboloid obtained by revolving the curve $y = x^2$, $0 \leq x \leq 3$, about the y-axis. $x$ and $y$ are measured in feet. The tank is initially full of water. Find the work needed to pump the water to a height 10 feet above the tank. The water weighs 62.4 pounds per cubic foot.
(4) [20 pts] Evaluate each of the following integrals.

(a) \( \int \frac{1}{(x^2 + 1)^{\frac{3}{2}}} \, dx \)

(b) \( \int \frac{2x}{x^2 - 5x + 6} \, dx \)

(Hint: use partial fractions)
(5) [20 pts] Evaluate each of the following integrals.

(a) \( \int \cos^3(x) \, dx \)

(b) \( \int x \sin(x) \, dx \)
(6) [20 pts] Determine if the following improper integrals converge and, if they do converge, evaluate them.

(a) \[ \int_{1}^{2} \frac{1}{\sqrt{x-1}} \, dx \] 

(b) \[ \int_{-\infty}^{\infty} x^2 e^{-x^4} \, dx \]
(7) [10 pts] Does the series \( \sum_{n=2}^{\infty} \frac{1}{n(n \ln n)^2} \) converge or diverge? Give reasons for your answer!
(8) [10 pts] Determine whether this series converges absolutely, converges conditionally, or diverges. Give reasons for your answer! \[ \sum_{n=1}^{\infty} \frac{(-1)^n \cdot 23}{2n + 3} \]
(9) [15 pts] Find the interval of convergence of \( \sum_{n=1}^{\infty} \frac{2^n}{n^2} x^n \). Does it converge absolutely, converge conditionally or diverge at the endpoints of the interval of convergence?
(10) [10 pts] Compute the first three non-zero terms in the Maclaurin expansion of
\( f(x) = x + e^{-x} \sin x \)