1. For a positive integer $n$, denote by $P_n$ the vector space of real polynomials of degree at most $n - 1$.
   (a) Given a real number $\alpha$, prove that the polynomials $1, x - \alpha, (x - \alpha)^2, \ldots, (x - \alpha)^{n-1}$ form a basis for $P_n$.
   (b) Find the coordinates of the polynomial $f(x) = a_0 + a_1 x + \ldots + a_{n-1} x^{n-1}$ with respect to the basis from (a).

2. Let $\lambda$ be an eigenvalue of a linear transformation $\sigma$ of a finite-dimensional vector space $V$ over a field $F$, $\sigma : V \rightarrow V$. Denote by $S(\lambda)$ the set of all eigenvectors of $\sigma$ corresponding to $\lambda$, together with the zero vector. Prove:
   (a) $S(\lambda)$ is an invariant subspace of $\sigma$.
   (b) $\dim S(\lambda)$ does not exceed the multiplicity of $\lambda$ as a root of the characteristic equation of $\sigma$.

3. Let $A$ be a complex matrix with characteristic polynomial $(x + 3)^4 (x - 8)^3$ and minimal polynomial $(x + 3)^3 (x - 8)^2$. What is the Jordan form for $A$?

4. Let $\alpha, \beta$ be vectors in a finite-dimensional vector space $V$ over a field $F$.
   (a) If $\alpha \neq 0$, show there exists a linear functional $\phi$ satisfying $\phi(\alpha) \neq 0$.
   (b) Prove. If $\phi(\beta) = 0$ implies $\phi(\alpha) = 0$ for all linear functionals $\phi$, then $\alpha = c\beta$ for some $c \in F$.

5. (a) Determine whether the real quadratic form $q(x, y, z) = x^2 + 3y^2 + 4z^2 + 4xy - xz$ is positive definite.
   (b) Can the polar form of $q(x, y, z)$ be used to define an inner product on the 3-dimensional real vector space $\mathbb{R}^3$? Explain.

6. Let $\phi$ be a normal transformation of a finite-dimensional unitary space, and let $\alpha$ be an eigenvector of $\phi$ corresponding to an eigenvalue $\lambda$. Prove $\alpha$ is an eigenvector of $\phi^*$ corresponding to the eigenvalue $\bar{\lambda}$.