Topics for Qualifying Exam in Complex Analysis

I Complex Plane and Elementary Function.
   a) Complex Numbers
   b) Polar Representation
   c) Stereographic Projection
   d) The Square and Square Root Functions
   e) The Exponential Function
   f) The Logarithm Function
   g) Power Functions and Phase Factors
   h) Trigonometric and Hyperbolic Functions

II Analytic Functions
   a) Review of Basic Analysis
   b) Analytic Functions
   c) The Cauchy-Riemann Equations
   d) Inverse Mappings and the Jacobian
   e) Harmonic Functions
   f) Conformal Mappings
   g) Fractional Linear Transformations

III Line Integrals and Harmonic Functions
   a) Line Integrals and Green’s Theorem
   b) Independence of Path
   c) Harmonic Conjugates
   d) The Mean Value Property
   e) The Maximum Principle

IV Complex Integration and Analyticity
   a) Complex Line Integrals
   b) Fundamental Theorem of Calculus for Analytic Functions
   c) Cauchy’s Theorem
   d) The Cauchy Integral Formula
   e) Liouville’s Theorem
   f) Morera’s Theorem
   g) Goursat’s Theorem
   h) Complex Notation and Pompeiu’s Formula

V Power Series
   a) Infinite Series
   b) Sequences and Series of Functions
   c) Power Series
   d) Power Series Expansion of an Analytic Function
   e) Power Series Expansion at Infinity
   f) Manipulation of Power Series
   g) The Zeros of an Analytic Function
h) Analytic Continuation

VI Laurent Series and Isolated Singularities
   a) The Laurent Decomposition
   b) Isolated Singularities of an Analytic Function
   c) Isolated Singularity at Infinity
   d) Partial Fractions Decomposition

VII The Residue Calculus
   a) The Residue Theorem
   b) Integrals Featuring Rational Functions
   c) Integrals of Trigonometric Functions
   d) Integrands with Branch Points
   e) Fractional Residues
   f) Principal Values
   g) Jordan’s Lemma
   h) Exterior Domains

VIII The Logarithmic Integral
   a) The Argument Principle
   b) Rouche’s Theorem
   c) Hurwitz’s Theorem
   d) Open Mapping and Inverse Function Theorems

IX The Schwarz Lemma and Hyperbolic Geometry
   a) The Schwarz Lemma
   b) Conformal Self-Maps of the Unit Disk

X Harmonic Functions and the Reflection Principle
   a) The Poisson Integral Formula
   b) Characterization of Harmonic Functions
   c) The Schwarz Reflection Principle

XI Conformal Mapping
   a) Mappings to the Unit Disk and Upper Half-Plane
   b) The Riemann Mapping Theorem
   c) Compactness of Families of Functions
   d) Proof of the Riemann Mapping Theorem

References: Complex Analysis by T.W. Gamelin