Do all five problems as directed. All answers are to be supported by proofs and/or reasoning.

\( \nabla \) Do any two of (a), (b) or (c).

\( \nabla (a) \) Let \( f_i: X_i \rightarrow Y_i, i = 1, 2 \) be continuous maps. With respect to the product topologies, show that \( f_1 \times f_2: X_1 \times X_2 \rightarrow Y_1 \times Y_2, (f_1 \times f_2)(x_1, x_2) = (f_1(x_1), f_2(x_2)) \) is continuous.

\( \nabla (b) \) Let \( \prod_\alpha X_\alpha \) denote the product space with the product topology, and let \( A_\alpha \subset X_\alpha \) for all \( \alpha \in J \).

\( \times \) Show that \( \prod_\alpha \overline{A_\alpha} = \prod_\alpha \overline{A_\alpha} \)

\( \checkmark (c) \) Show that the diagonal function \( \Delta: \mathbb{R} \rightarrow \mathbb{R}^\omega \) is continuous with respect to the product topology, and not continuous with respect to the box topology, into the countable product of the real line.

2. Do all three parts.

\( \checkmark (a) \) Show that a continuous surjection \( p: X \rightarrow Y \) is a quotient map if for every space \( Z \) and each function \( g: Y \rightarrow Z, g \) is continuous whenever the composition \( g \circ p \) is continuous.

\( \checkmark (b) \) Is the projection of the \( xy \)-plane onto the \( x \)-axis a quotient map?

\( \checkmark (c) \) Recall that there is the known criterion for asserting when a function \( Z \rightarrow \prod_\alpha X_\alpha \) into a product space is continuous. State and prove a corresponding assertion for (certain) functions out of a quotient space \( Y \) (\( p: X \rightarrow Y \) a quotient map) to be continuous.

\( \checkmark \) Recall that a space \( X \) is locally compact at \( x \) if there is a compact subspace \( C \) of \( X \) that contains a neighborhood (i.e. an open set) of \( x \). Show for a Hausdorff space that is locally compact at \( x \), that for each neighborhood \( U \) of \( x \) there is a neighborhood \( V \) of \( x \) such that \( \overline{V} \) is compact and \( \overline{V} \subset U \).

\( \times \) Do either (a) or (b).

\( \checkmark (a) \) Show if \( X \) is normal, every pair of disjoint closed sets has neighborhoods whose closures are disjoint.

\( \times (b) \) Show if \( X \) has the order topology, \( X \) is regular.

5. Do either (a) or (b).

\( \nabla (a) \) Let \( X \) be compactly generated space and let \( (Y, d) \) be a metric space. Show that the space of continuous functions \( C(X, Y) \) is closed in \( Y^X \) in the topology of compact convergence. (Recall that a function \( f \) in this topology is continuous if its restriction to each compact set \( C \) is continuous.)

\( \nabla (b) \) Let \( X \) be locally compact Hausdorff and let the space of continuous functions \( C(X, Y) \) from \( X \) to \( Y \) have the compact-open topology. Show the function \( e: X \times C(X, Y) \rightarrow Y, e(x, f) = f(x) \) is continuous. (Suggestion: begin with any open set in \( Y \) and use that \( f \) is continuous and \( X \) is locally compact Hausdorff.)

\[ e(x, f) = f(x) \]